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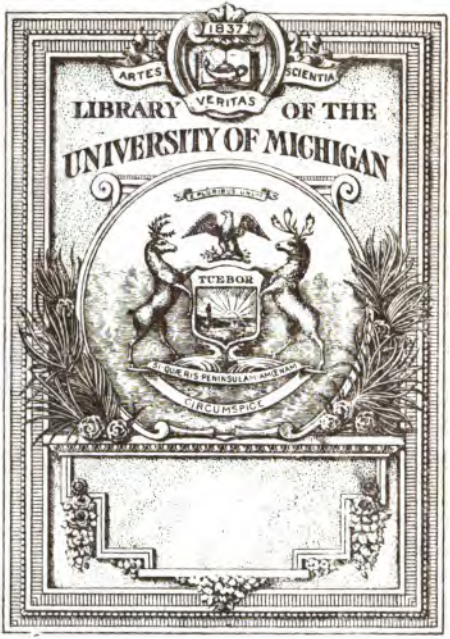
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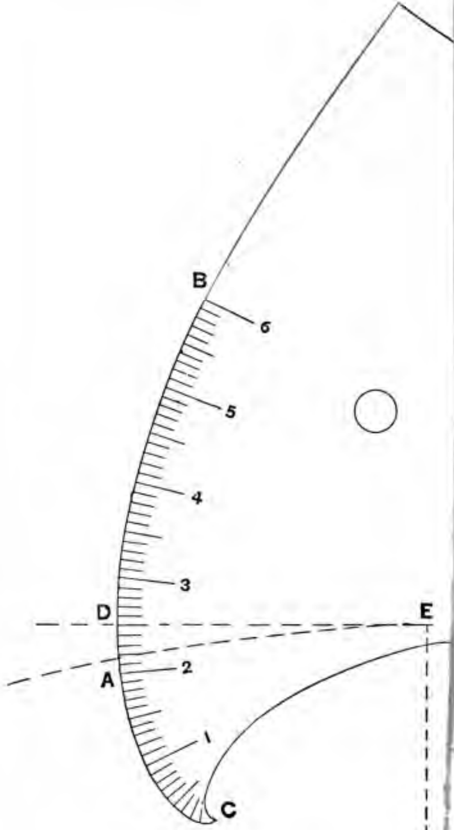
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# PRACTICAL TREATISE

ON THE

# TEETH OF WHEELS

With the Theory and the use of

## ROBINSON'S ODONTOGRAPH.

Stillman Williams

BY

**S. W. ROBINSON,**

*Professor of Mechanical Engineering,*

OHIO STATE UNIVERSITY.

**REVISED, WITH ADDITIONS.**

ILLUSTRATED.

NEW YORK:
D. VAN NOSTRAND, PUBLISHER,
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1888.

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## Preface to the Second Edition.

The first edition of this little volume resulted from an effort to place in the hands of machine builders simple and improved facilities for obtaining correct gearing, and that the effort was appreciated to some extent is evinced by the fact of the exhaustion of that edition.

In the present edition improvements and additions have been made for the purpose of the better securing the original object, all tending to give the subject matter a more practical bearing.

Correct gear teeth are essential for the economic transmission of power, as well as extremely desirable for avoiding needless rattling. Many of the cast gears introduced into mills and factories may properly be termed rattling machines, and simply because the designer had too vague a conception of the theory of gearing, or because he sought to avoid the trouble of laying them out.

If this manual, and the odontograph, shall contribute toward relieving these inconveniences, the object sought, when the instrument was devised, and an effort made to present it in convenient form for use, will be realized.

The Templet Odontograph described in the second part received an award at the Centennial Exposition of '76, where it was first presented to the public, a testimonial which its somewhat extended use has since justified. A new and enlarged table, for use in connection with the instrument, is given in this edition, and substituted for several tables of the former edition, a table which covers all cases of epicycloidal and involute gearing; but to render the use of the instrument most convenient in practice, numerous tables are furnished with the instrument to cover special systems of gearing, but which tables are not of sufficient interest to this volume to be included in it.

S. W. R.

OHIO STATE UNIVERSITY,
April 4th, 1888.

# PART I.

## ON THE
## THEORY OF GEAR TEETH,

### AND ADAPTATION OF PARTICULAR FORMS FOR CERTAIN PURPOSES.

THOUGH the curves which may be employed for the outlines of mathematically correct gear teeth are infinite in number, varying greatly in external appearance, yet all of them must conform with one general principle, viz.: *tooth curves must transmit, from one shaft to the other, the same motion as would the pitch lines when acting by simply rolling contact.*

This truth is, in reality, axiomatic; and for the present purpose needs no discussion or comment. It forms the basis of the present investigation, the object of which is to show how the curves may differ, and the adaptation of certain ones for special purposes.

In this treatise the terminology indicated below is adopted.

The *pitch lines* of a pair of wheels are lines which pass near the mid-height of all the teeth, and separate the faces from the flanks. These lines must always exist, whether the wheels are designed with regard to them or not, and on which the tooth spaces of one wheel equal those of the other.

The *face* of a tooth is that part of the outline of one side of a tooth which extends from the point or top of tooth to the pitch line.

The *flank* of a tooth is that part of the outline of one side of a tooth which extends from the pitch line to the bottom of the space between two adjacent teeth. The length of this, reckoned in the direction of a radius, is usually made a little greater than the length of the face, similarly measured, to allow for *clearance.*

The *pitch* is the distance, reckoned

on the pitch line, from a point on one tooth to the corresponding point on the next.

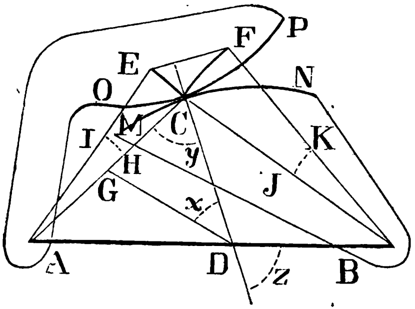
The *velocity-ratio* is the ratio of the angular velocities of the wheels, and may be constant or variable. If constant, it is equal to the number of revolutions per minute of one wheel, divided by the same for the other.

In the study of gearing two kinds of contact are considered, viz.: rolling contact and sliding contact. The former takes place between the pitch lines, and the latter between the curves forming the outlines of the teeth. When the pitch lines are not circular, the problem of determining them so as to give true rolling contact, with fixed axes, is quite extensive. The present paper is confined to the outlines of teeth, the pitch lines being supposed given.

In the figure, let A and B represent the centers of a pair of gears, and M N and O P the curves of a pair of teeth in action. The point of contact C is chosen at one side of the line of centers, so as to represent the general case.

Now, suppose the wheel A to swing through the small angle C A E. The

FIG. 1.



point C will describe the elementary arc C E. At the same time the wheel B, to maintain contact with A, must move through the elementary angle C B F, and C through the arc C F.

To determine C F, it is sufficient to note that as the movements of the wheels are elementary, the common tangent to the curves at C will scarcely change its inclination appreciably. Therefore, the end of the radius B C will be found at F, in a line, E F, parallel to the common tangent.

Draw a common normal C D to the curves. It will be perpendicular to E F. Then the velocity-ratio may be obtained as follows:

At a unit's distance from A and B draw H I and J K respectively. These arcs will serve to measure the angular velocities of the wheels, and hence:

$$\text{Velocity-ratio}=\frac{HI}{JK}.$$

Draw D G parallel to B C.

The triangle C E F is similar to C D G, and hence:

$$\begin{aligned}
&AH : AC :: HI : CE\\
&BJ : BC :: JK : CF\\
&CE : CF :: CG : DG\\
&DG : BC :: AD : AB\\
&CG : AC :: BD : AB
\end{aligned}$$

Whence, observing that $AH=BJ=$ unity,

$$\text{Velocity-ratio}=\frac{HI}{JK}=\frac{EC}{AC}\cdot\frac{BC}{CF}=\frac{CG}{DG}\cdot$$

$$\frac{BC}{AC}=\frac{AB}{AD}\cdot\frac{BD}{AB}=\frac{BD}{AD}=\frac{V^{*}}{v},$$

---

[*] This geometrical solution is essentially the

if $V$ and $v$ represent the angular velocities of A and B.

Hence, when rotary motion is transmitted from one shaft to another by sliding contact as in gear teeth, the angular velocities are to each other inversely as the segments, or parts of the line of centers, formed by its being cut by the common normal to the curves.

It is to be observed that at the end of

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same as that given in Belanger's most excellent work on Cinematique, page 90, where also the velocity of sliding is found to be proportional to C D.

Those familiar with trigonometry may prefer the following trigonometrical solution. We have

$$CE = AC . V, \; CF = BC . v,$$

$$CEF = \text{angle } y, \; CFE = x, \; ADC = z;$$

and hence:

$$CE \sin. y = CF \sin. x,$$

$$AC \sin. y = AD \sin. z,$$

$$BC \sin. x = BD \sin. z,$$

or

$$V . AD \sin. z = v . BD \sin. z,$$

and

$$\frac{V}{v} = \frac{BD}{AD},$$

as before.

the movements C E and C F, the points which were in mutual contact at C are now removed a distance E F; and as this displacement was accompanied with slipping, the amount of the latter is E F. In the similar triangles C E F and C D G, we have the same relation between E F and C D, as we have between C E and C G, or C F and G D. Hence, E F can only be zero when C D vanishes; or when the point of contact is on the line of centers. For this, the contact has changed from sliding to rolling; and, therefore, for rolling contact, the angular velocities are inversely as the radii of contact. In circular wheels the velocity-ratio must remain constant, because these segments are constant; whereas, in non-circular wheels it is variable. In the former, the point of contact remains fixed, while, for the latter, its position varies along the line A B.

The contact of a pair of teeth must continue for a time, at least, long enough for the next pair of teeth to come fairly to contact; or, long enough for the pair of teeth to move through a space as great as the length of the pitch. This contact cannot therefore be confined to the line of centers, and hence sliding of the curves upon each other is unavoidable.*

From the preceding discussions, it follows that in all gear wheels, circular or non-circular, the common normal to the sliding curves, and the pitch lines, must both maintain a common point of intersection with the line of centers: and this condition is the only one necessary for fulfilling the general principle stated at the outset. The common normal, because it is the line of action of the pressure between the teeth, friction neglected, and for its importance otherwise, is called the *line of action*.

Suppose, for example, any curve whatever, as *ae* Fig. 2, be taken as the

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* [illegible] may be [illegible] But this is [illegible] of teeth [illegible] of [illegible]

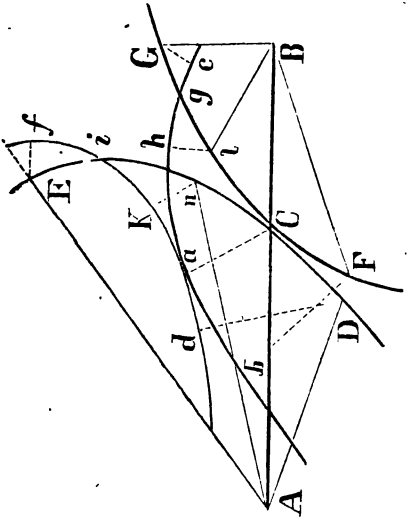

Fig. 2.

form of the side of a tooth of a gear wheel revolving about B. Then let the curve *d a f* be found for the side of a tooth of A, which, acting by sliding contact on *b a c*, will give to A the same angular motion as that transmitted by the rolling curves D E and F G, the latter being supposed circular or not. Let D and F be two points of the rolling curves which come to mutual contact on the line of centers, and similarly for E and G. Then arc C D = arc C F, and arc C E = arc C G, &c.

Now for the present point of contact C of the pitch lines there must be a point of contact of the tooth curves, and because the common normal to those curves must meet A B in C, we simply draw C *a* normal to *b a c*. This is the present point of contact of the tooth curves, and hence a point of the curve sought. To find another point, draw F *b* normal to *b a c*. Then draw D *d* equal in length to F *b*, and making the same angle with A D, that F *b* does with B F prolonged. Then, *d* is a second point of the required curve; because when D and F are in contact on the line A B, D *d* and F *b* will coincide in length and direction, with *d* and *b* superposed, forming the point of contact of the tooth-curves. In like manner *f*, and indeed any number of points may be found. The curve *d a f*, traced through these points, is the desired curve.*

In the above example, A and B are supposed to be the centers of rotation, in which case D E, and F G, must be a pair of correct rolling curves. But the solution is still more general. Any two curves whatever, without fixed axes even, may be used for the rolling curves, and the sliding curves determined. When radii do not exist, tangents to the rolling curves may be used to reckon the angles by. Such a case, however, would but rarely occur in practice.

This solution indicates that one tooth curve may be assumed, and the other formed as a dependence. It fails, however, as being both theoretically and mechanically impossible, and so with any theory, when the normals C *a*, F *b*, &c., will not strike the pitch line.

Though one tooth-curve may be assumed and the other determined, the theory admits of other solutions; the most common being by aid of *describing* curves, called also *generating* or *tracing* curves. In this case the describing curve is assumed and both tooth-curves made dependent. To show that this solution depends upon one and the same general theory, let us find a describing curve which will generate the tooth-curve *b a c*, or *d a f*.

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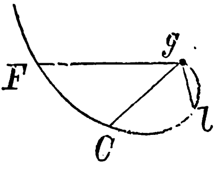
* This solution given in Belanger's Cinematique, p. 97, appears to be due to De la Hire; and given by him as early as 1694. See Willis' Princ. Mechanism, 2d ed., p. 89.

In Fig. 3 draw *g l* equal to *h l*, Fig 2. Then take *l* C, Fig. 3, equal *l* C, Fig 2. Also *g* C, Fig. 3, equal *a* C, Fig. 2. And again C F and *g* F, Fig. 3, equal C F and *b* F, Fig. 2, respectively, &c., for as many points as desired. Then trace the curve *g l* C F, Fig. 3. This is the describing curve sought. Now suppose this curve be placed in contact with F G, Fig. 2, so that *g*, Fig. 3, falls at *g*, Fig.

FIG. 3.



2. Then by rolling the describing curve along toward F, the points *l*, C, F, Fig. 3, will exactly fall at *l*, C, F, Fig. 2, because the arcs *g l*, *l* C, C F, are equal in the two figures. Also the construction makes *g l*, *g* C, *g* F, &c., intersect the arc of the describing curve at the same angles that the normals *h l*, *a* C, *b* F, &c., intersect the pitch line, which is easily seen to be true by supposing the points *g*, C, F, &c., very numerous. Hence the radii vectors of Fig. 3 will successively come to exact coincidence with the normals of Fig. 2, and at the same time *g* will trace out the very curve *g h a b*.* And similarly, by rolling the same curve from *i* to D, Fig. 2, the curve *i k a d* will be traced. In like manner a describing curve can be produced which will generate, by rolling from *g* or *i* toward G or E, the remaining portions *g c* and *i f* of the tooth-curves.

Thus it appears that a generating curve can always be found that will trace out a given tooth-curve. But whenever the latter are neither of them fixed, we are at liberty to assume a describing curve, and this may be of any form whatever which may admit of rolling on the pitch lines. The two solutions are therefore one and the same, the difference being simply in the part assumed. And within this are comprehended all known forms of gearing, such as involute, epicycloidal, conjugate, epitrochoidal, pin-gearing, Prof. Edward Sang's hour glass curve gearing, &c., &c.

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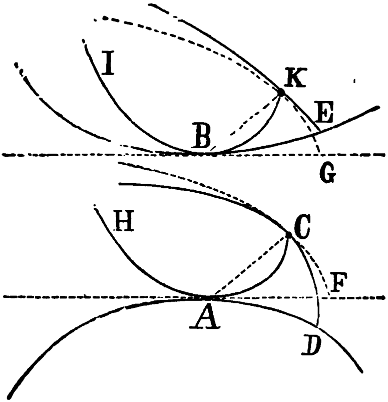
* De la Hire, in 1733.

The solution of the problem is therefore reduced to simply this: any describing curve rolled upon the outside of one pitch line, and upon the inside of the other pitch line, beginning at mutual contact points, will describe a pair of curves, a face and flank, which will work correctly.

This may be demonstrated by a direct graphical method as follows.

Take pitch lines A D and B E, circular or non-circular, and a describing curve C A H; or, which is the same, K B I. Also take D and E as the pair

FIG. 4.



of mutual contact points, where the pair of tooth curves sought begin. Let C A H roll in outside contact from D to A, describing D C by a tracing point C; and similarly rolling the same curve from E to B, describes the curve E K. This rolling may be continued to any extent, giving the full line curves D C, and E K produced.

If the same curve be rolled on tangents at A and B, two equal curves will be described, which are shown in dotted lines. Let arc D A = arc E B = arc C A = arc K B = A F = B G. Then chord A C = chord B K, and angle C A F = angle K B G. Hence if A and B be placed in tangential contact, the tangents and dotted curves will coincide precisely; and the full line curves, or tooth-curves, will have a point of tangency at C K. Also the curves will not pass, one to the opposite side of the other, because the full line curve for A lies wholly below, and for B wholly above the mutually coinciding dotted curves. This being true for one point taken at random must be true for all, giving the precise conditions required for tooth-curves.

We might suppose for illustration that the three curves roll upon each other with one point A, Fig. 5, in common tangency. The tracing point C would then trace, simultaneously, the two curves shown, one upon the plane of each wheel, which will serve to work correctly as a face and flank.

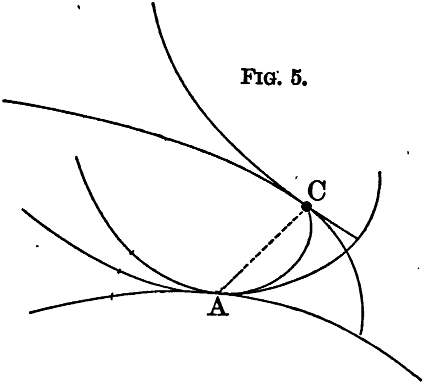


FIG. 5.

We are now prepared to draw the following conclusions:

1st. The faces and flanks all have their origins at the pitch line.

2d. Each face and flank couplet must be generated by rolling the tracing curve in the same direction, on the outside of one, and inside of the other pitch line.

3d. The contact of a couplet is confined to one side of the line of centers.

4th. The face and flank of a couplet must be described with the same describing curve as also all faces and flanks that interchange action during the working of the pair of gear wheels in question.

Different describing curves give different forms of gear teeth, some of the most important of which will now be considered.*

In practice the pitch lines are usually circular, though on shaping and slotting machines, and others requiring a "quick return," non-circular wheels have been employed with advantage. In Fig. 6 is given such a pair of non-circular wheels, drawn carefully to a scale, with each tooth-curve worked out for its particular position by the general theory.† The describing curve used was a circle, the diameter of which was about one-third that of the pitch line of A. These wheels were designed with special reference to

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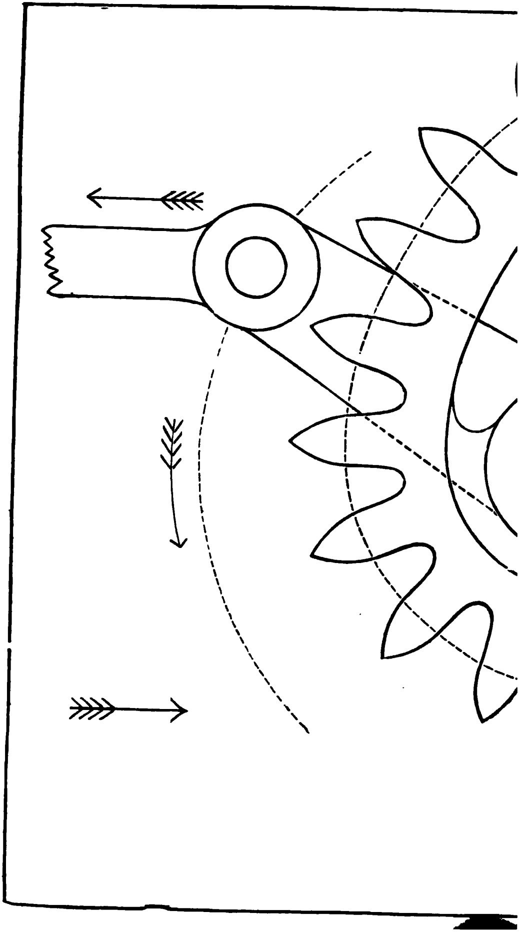
* It may be observed by comparing Figs. 2, 3 and 5 that the generating curve must lie on the opposite side of the pitch line from where the intersections of the normals are found. From this it follows that if these intersections cross the pitch line as the rolling proceeds, so must the describing curve. The latter will then change its direction with a cusp.

† Drawn by a former pupil, Mr. Emory Patch.



adaptation for shaping and slotting machines, in which the tool is reciprocated by a crank and pitman. The forward stroke is uniform, except for one-fourteenth at each end, given for slowing down for passing the center. The return is accomplished in one-half the time the entire forward stroke requires. The teeth are made large, proportionately, for the purpose of illustrating the fact that such wheels require teeth which differ from each other in form.

Circular wheels form only a special case of the general theory; but as these are of almost exclusive use in practice, with the teeth varied for different purposes, the following important classes will be considered separately, viz.:

I. Wheels with epicycloidal teeth;
II. Wheels with involute teeth;
III. Wheels with conjugate teeth.

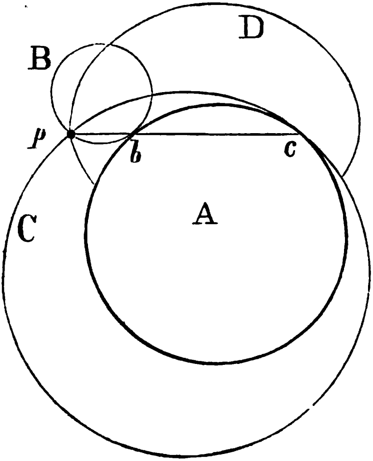
Epitrochoidal curves have been suggested for tooth-curves, but as they do not admit of tooth-contacts near the line of centers, where most desirable, they are not useful for general purposes.

## I. EPICYCLOIDAL GEARING.

This name is applied because the tooth-curves are epicycloids and hypocycloids. An epicycloid is traced by a point upon the plane of a circle, when that point is guided by being attached to the circumference of a second circle which rolls upon the first. If rolled external, it is an epicycloid; if internal an hypocycloid.

A curious fact relating to these curves deserves notice, viz.: that every epicycloid and hypocycloid admits of being generated by two different rolling circles. To illustrate, suppose A *b* *c* to be a primitive or pitch circle, and B a rolling or describing circle, tracing with *p* the epicycloid D, by rolling on A. Draw the line *p* *b* *c* through the point of tangency *b*. Then *b* *p* will be a normal to the epicycloid at *p*, because an elementary part of the latter is described while *b* serves as an instantaneous axis for the wheel B to turn about. Now, this elementary portion of D will be the same if *c* be the center instead of *b*. This being true for each elementary part of D, it follows that a circle C, tangent at $c$, and having a tracing point $p$, will trace the same epicycloid that B will. It can be easily shown that the difference of the diameters of C and B is equal to the diameter of A.*

FIG. 7.



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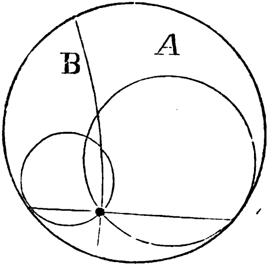
* Due to Duhamel, see Bour Cours de Cinematique, p. 120.

Fig. 8 indicates how this double generation applies to the hypocycloid, in which A is the pitch circle and B the hypocycloid. The sum of the two diameters of the generating circles evidently equal the diameter of the pitch circle.

Fig. 8.



That the teeth be mechanically possible, it is necessary that the generating circles be as small, or smaller than the pitch circles within which they roll; otherwise the teeth would be much undercut.

This range in the relation of sizes gives variety of form of teeth, the following of which are the most essential:

1st. Flanks radial.
2d. Flank parallel, for strength.
3d. For minimum of friction.
4th. For change-gears and sets.
5th. Pin gearing.

## 1ST. EPICYCLOIDAL TEETH WITH RADIAL FLANKS.

Suppose a generating circle to roll inside a pitch circle, with the diameter of the former equal to the radius of the latter. By reference to Fig. 8 we see that the two generating circles would be equal, and the hypocycloid a straight line, a diameter of the pitch circle. This principle is the foundation of the well-known White's parallel motion.

Hence when the diameters of the generating circles equal the radii of the pitch circles in which they roll, the flanks will be radial, and very convenient to construct. In practice, therefore, we have simply to delineate the epicycloids, with circles half as large as the opposite pitch circles, and strike in the tangent radii for flanks. This form of gearing is the most convenient for practical execution of any good form known, and for this reason is the form usually adopted, when correct delineation is attempted. Indeed, many designers have no idea of the existence of any other correct form, except, possibly, the involute.

FIG. 9.

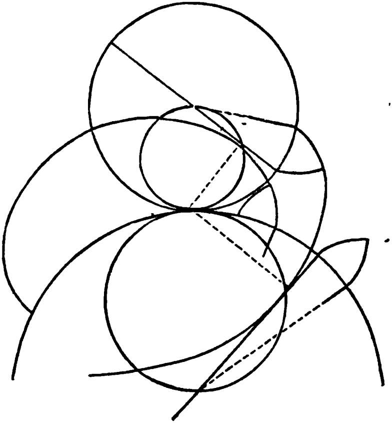


Fig. 9 represents the simultaneous generation of the faces and flanks for both wheels, the four circles being supposed to roll upon each other with only one point in common, each tracing point describing two lines, a face for one wheel and a flank for the other.

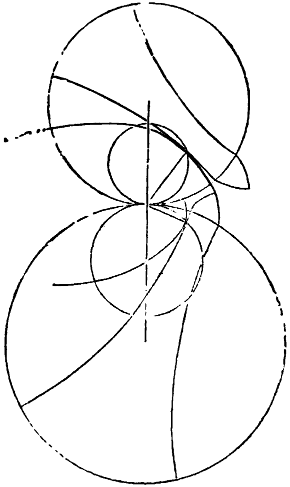
In wheels of few teeth, the latter appear somewhat undercut, and weakened, when the flanks are drawn wholly radial.

But it is easily demonstrated that a face works upon only a small portion of the flank, which also is adjacent to the pitch line. Below this the teeth may be thickened by clearance curves, almost entirely removing this objection. An advantage is gained by these teeth which probably more than compensates for the above objection. This consists in the great freedom from crowding of the wheels from each other, due to the near approach of the line of action to the perpendicular to the line of centers; and this is evidently necessary for most favorably transmitting the rotative forces from one wheel to the other. For instance, if the line of action be made parallel to the line of centers no rotative effect is secured.

## 2D. FLANKS PARALLEL.

By employing smaller generating circles than in the above, the flanks may be rendered more nearly parallel, as shown in Fig. 10; and the bases of the teeth thickened as required for strength. Clearing curves may still be introduced for further strengthening of the teeth.

FIG. 10.



No rule of proportions is prescribed; in each case the diameters of the rolling circles must be determined by the judgment of the designer.

The flanks are only approximately parallel, because, in reality, they are the slightly curved hypocycloids. But for that small part against which the faces act, a straight line may be substituted without sensible error.

The rolling circles being smaller than for the case of radial flanks, the epicycloids will be smaller for the same pitch lines, and hence the points of the teeth narrower. This results in a greater general inclination of the side of a tooth to a radius, causing greater inclination of the line of action from the perpendicular to the line of centers, inducing increased lateral crowding, and consequent friction. In a long train of gearing it is therefore doubtful whether this form of tooth is any safer than the preceding, because with greater strength there is greater resistance.

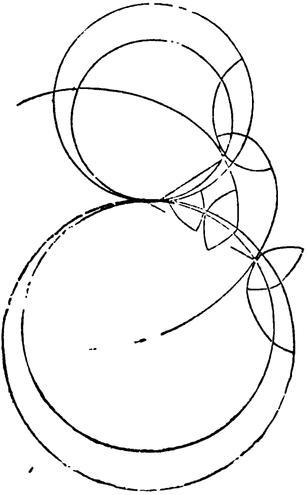
### 3D. FOR MINIMUM OF FRICTION.

From the preceding examples, it appears that the friction is reduced by approaching the line of action to the perpendicular to the line of centers. The line of action in epicycloidal teeth being the chord to the generating circle which connects the describing point with the point of rolling, it appears that the most favorable condition is secured by making the generating circles large as possible.

Again the generating circles, when situated as in Figs. 9, 10 and 11, form the path of the points of contact in their journey from the point of beginning, to the point of quitting contact, one circle for one side and the other for the opposite side of the line of centers; so that large circles, in both senses, appear most favorable.

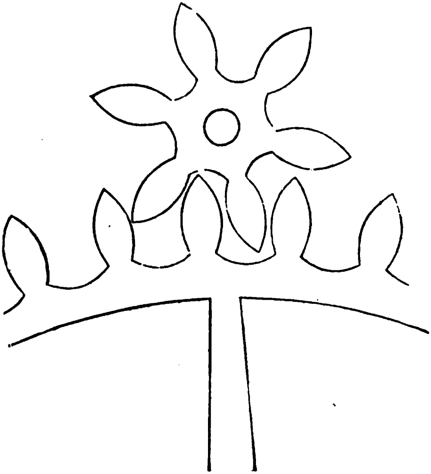
Fig. 11 presents these conditions, and indicates that the teeth are very much undercut; indeed, in extreme cases, entirely cut off, by reason of the convex flanks, due to the excessively large rolling circles.

FIG. 11.



It is evident that such teeth cannot be used where great strength is required, as in mill-work; but where the resistance is slight, and smoothness of motion and freedom of action are requisites, as in horology, this form may be used with great advantage. Fig. 12 is a careful drawing of such a pair of wheels, with teeth much undercut.

Fig. 12.



4th. CHANGE GEARS, AND SETS.

The most common use of change gears occurs with the machinist's engine lathe for screw-cutting. In this, and most other cases requiring such wheels, the teeth are small; and when only approximate, occasion no especial inconvenience. But sets sometimes occur in mill gearing where correct forms of teeth are necessary.

For such cases it is only needful that *one* generating circle be employed for the whole set.* The circle should not be more than half as large as the smallest wheel of the set. The teeth of the latter will then have radial flanks, while those of all the others have concave flanks and thick bases.

### 5TH. PIN GEARING.

By increasing the generating circle, Fig. 11, the hypocycloid decreases until, when the former coincides with the pitch circle, the latter becomes a point, the tracing point itself. Then the theoretical teeth are points for one wheel, and for the other epicycloids generated by the opposite pitch circle as shown in Fig. 13. In practice the points are enlarged to pins of cylindrical form, and the teeth reduced to lines parallel to the epicycloid and distant from them a space equal to the radius of the pin. Such wheels are sometimes called pin gearing.

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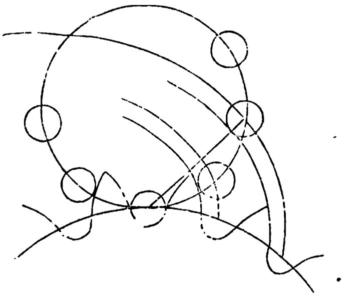
* This solution was first given by Prof. Willis, see Princ. of Mech., 2d ed., pp. 87, 118, and 186, the latter as in connection with the Willis Odontograph.

Fig. 13.



These wheels should work smoothly, because the line of action is suitably situated for a minimum of friction, but they are not free from objection for having the tooth contacts all on one side of the line of centers. These one-sided contacts serve best when the pin wheel is driven, and in this way this gearing is much used in modern clock-work. The pinion, called *wallower*, *trundle* or *lantern wheel*, is usually made for clocks by inserting wires between two collars; while the wheels are cut from plates, many of which are stacked and cut at once.* In this way should the wheel be quite thin, and wabble, the trundle may have sufficient length of pins to accommodate. Such gearing is cheaper than thick cut gearings, and may explain its introduction into the inexpensive, though very good, American clocks.

But it is probable that the very best conditions for clock and watch gearing would be secured by a sort of *cross* between Figs. 11 and 13, represented in Fig. 14. The generating circle should be so large that the hypocycloid, for both pitch lines, spans a breadth just sufficient for the tooth thickness, and the whole hypocycloid taken as that part of a tooth, or pin, which lies within the pitch line. The balance of the tooth

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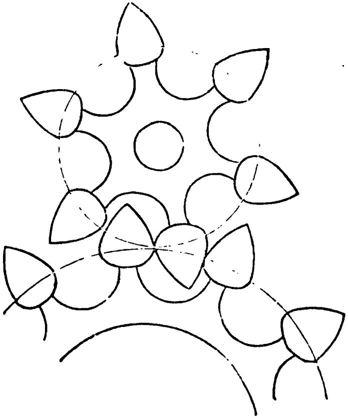
*In the full sense of the term *pin gearing*, the wheel teeth are also pins set into the edge of a disk.

should then extend outward, to a point, similarly as in Fig. 11. Thus contacts

FIG. 14.



will be secured on both sides of the line of centers, as required, with the nearest approach of the line of action to the perpendicular to the line of centers consistent therewith.

These wheels could be constructed by inserting suitably formed pins by one end into disks, and projecting toward each other from the two wheels; or, by supporting the teeth upon slender necks as shown, which is possible, because only small parts of the hypocycloids are brought into contact with the faces.

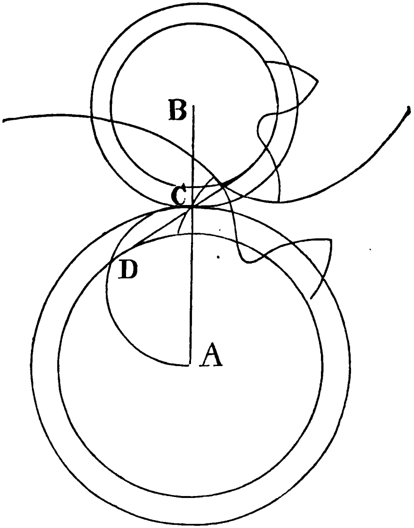
It is plain that if the line of action could be straight, and perpendicular to the line of centers, the wheels could be approached or withdrawn without causing great changes of conditions of action of the teeth. The form, Fig. 14, with teeth supported upon clearance curve-necks, would, therefore, be one of the best for this purpose, and nearly the same as found on the rollers of clothes-wringing machines, &c., as arrived at, probably, by the demands of practice.

## II. INVOLUTE TEETH.

The involute curve is a special case of the epicycloid in which the generating circle is infinite.

Let A and B represent two pitch circles in contact at C. Draw any inclined right line through C and two circles tangent to it with centers A and B. These are taken as the base circles for involutes. The ratio of the radii of these circles will be equal to that of the pitch circles. Suppose the inclined line,

FIG. 15.



C D, be produced, and wound upon the base circle A; and also separated near C, with a tracing point attached. As the string is unwound, the tracing point will describe an involute which may be taken for the side of a tooth of A as shown. A similar unwinding from B will describe the involute tooth-curve for it. It is plain that the contacts will all lie on the inclined line C D, because it will cut normally all the involutes described on the bases A and B. This is at once the line of action, and locus, or path, the points of contact.

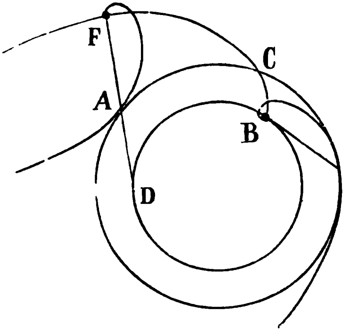
In the present case the line of action is inclined more or less, at pleasure, but must always pass C. This change of inclination may follow from varying the diameters of either the pitch or base circles. The former corresponds to changing the distance between the axes of the wheels, which these gears are well known to admit of, and work correctly. But when the distance is considerable, the teeth become monstrous thick at base, and objectionable; and unsuited for such purposes as clothes-wringers. The contact for these teeth is only possible between the points of tangency of the line of action with the base-circles. Hence, by enlarging the base-circles, these limits are constricted, and, finally, vanishing when the line of action becomes perpendicular to the line of centers, giving no contact. The best condition is a matter of judgment, and the designer must choose between objectionably few contacts, and objectionable inclination of line of action and thick teeth.

The contact being confined to the line of action, limited as above, it is plain that no bearing can occur upon a tooth below or inside the base circle, and usually clearance curves are arbitrarily struck in to give the necessary depth for receiving the points of the teeth of the other wheel.

One fact is worthy of note, serving to put these wheels in comparison with the epicycloidal. If a circle be struck with A C as diameter, it will always pass the point of tangency D. That point, in involute gearing, is the limit of contact of teeth; but not in epicycloidal gearing with radial flanks, though that will be one position. Intermediate between D and C the contact for involutes is on the line C D; and for epicycloids, on the arc C D. The line of action in the latter case will vary from a perpendicular to A B, to the inclination C D, while the point of contact moves from C to D. Hence, the average inclination of the line of action between C and D is much more favorable in epicycloidal gearing than involute, from which it appears that the former is the perferable form of gearing.

Fig. 15 may appear to furnish an example which is inconsistent with our general theory, because the generation is effected by rolling the describing curve or line on the base circle instead of the pitch line. But this is reconciled thus: In Fig. 16 take a pitch circle A C, base circle B D, describing line D F, with describing point F, and a log. spiral F A. Now as the describing line unwinds, tracing the curve C F, the line will cut the pitch circle at various points A at a constant angle, which follows from the concentricity of the pitch and base circles. As the radius vector makes a constant angle with the arc of a log. spiral if that angle at A be the same as D F with the circle at A, then the radius vector F A will coincide with the line D F

FIG. 16.



continually, and the tracing point will describe one and the same curve, whether carried by the spiral F A or line F D. This curve begins at C. To supply the remaining part C B, roll the same spiral inside as shown. This description will stop at B for the reason that the log.

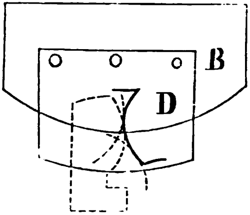
spiral then bears on the inside of the circle where its curvature equals that of the circle. Hence this is explained by the general theory, and the wheel has true flanks and faces, separated by the pitch line, as usual.

### III. CONJUGATE TEETH.

These might be styled *conjugate teeth*, becuse a tooth-curve for one wheel is drawn at random, or at least arbitrary, and the form of tooth for the other wheel found from it. The solution on the drawing board is the same as Fig. 2, and hence, it is in keeping with the general theory of the teeth of wheels, so that the forms of the teeth are mutually dependent upon each other, for which reason the teeth may properly be termed *conjugate teeth*. It would need no further consideration except for the exceedingly advantageous practical process for obtaining the second curve, first made known by Prof. Willis in 1837.

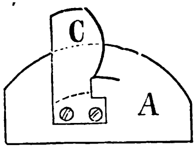
Let A and B represent the pitch lines, formed by cutting strips to the right curves. A tooth curve C is cut at pleasure, and mounted on A so that the card board D may play between A and

FIG. 17.



C. The card board D is secured to B. The pitch lines are then placed in contact, as shown by the dotted lines, and a line

FIG. 18.



traced on D, along the edge of C. Then A is rolled on B, without slipping, to a new position, and a second line traced by the edge of C. Thus proceed for various positions. An enveloping curve traced on D will be the required tooth-curve for B.

This process would apply admirably for matching a gear already existing, and no matter if it be worn, provided it is regular.

### SPACING OF GEAR TEETH.

In the construction of gearing, it should be observed that the uniform distribution of the teeth is very important for quiet working gearing, as well as the form of the teeth, and that a pair of cast gears will be noisy, whether from the one defect or the other.

Heavy cast gears with large and apparently very strong teeth made considerably in excess of strength for the purpose of safety, have been known to drop their teeth while in the intended service, and without any good reason, as would at first appear. But after some puzzling inquiries it was found that the shocks upon the teeth by their striking each other, when in irregular spacing, or form, has caused strains sufficient to knock off the teeth. For instance, suppose an error of 1-100 of an inch in the position of a tooth of a gear 2 feet in diameter. running at 240 revolutions per minute, and suppose the portion of the gear affected by the shock weighs 100 lbs., then the pressure upon the tooth due to the shock will be about 11,000 lbs., a force that would tear off quite a gear tooth.

### PRACTICAL OPERATIONS.

In actually executing wheel-work, there are certain operations which contribute to the accuracy and dispatch. The draughtsman can draw his tracing curve in several positions, and by stepping with spacing dividers arrive at several points in the desired tooth-curves. But in thus measuring off distances on arcs, errors occur, in that the dividers measure chords instead of arcs. Also each placing of the divider points involves a small error, depending upon eyesight, precision of draughtsman, &c. Again several operations, following and depending upon each other, between the theory and the result, entail cumulative errors, sometimes so great that the result obtained scarcely resembles the truth. The fewer the intervening operations, the less will the truth be adulterated with error.

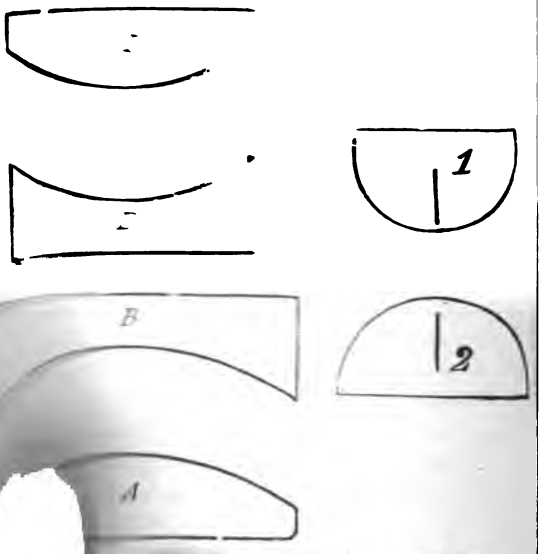
Templets are valuable because they secure a more direct relation between the theory and final tooth curves. In some instances a still closer relation has been obtained. To illustrate, the templets give the curve and then the teeth or cutters are to be formed from it by an eye-and-hand process. But in some watch-works, epicycloidal machines are used in which the cutters are formed direct from the rolling curves. These cutters, however, are *master cutters* from which the working cutters are formed. But this last operation admits of great precision.

In Fig. 17, the pieces A and B are called pitch templets, and a piece cut to the form of the describing curve, as in Fig. 3, is called a describing templet.

By a set of such templets the operation represented in Fig. 4 may be undertaken. This method of laying out tooth-

[illegible] describing templets, is [illegible] described and illustrated, for circular gears. The object is to show the number of templets required and their practical use for one pair of gears.

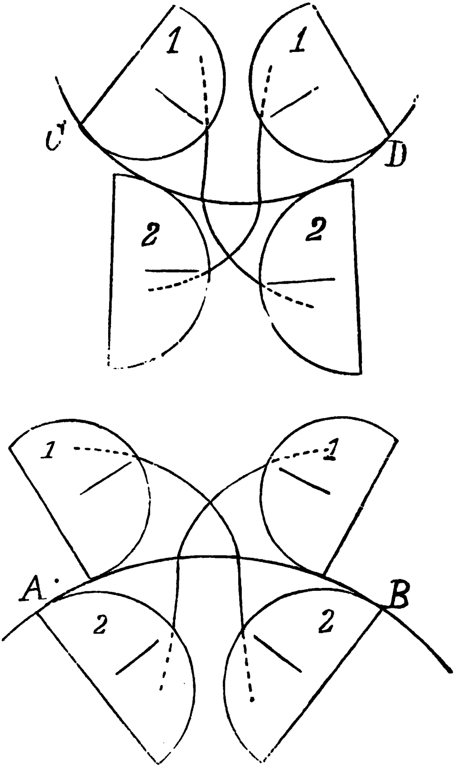
Fig. 18.



[illegible] Fig. 18 are shown all the templets [illegible] wing the complete tooth profiles shown in Fig. 19. A, B, C and D are called *Pitch Templets*, because they fit the pitch lines A B and C D. They may be cut out of thin wood and carefully dressed to the pitch line circles. The smaller circle arcs, 1 and 2, are also templets cut from thin wood, and to be used in describing the curves of the teeth by rolling them along the pitch lines, while attached tracing points mark the curves. These are called *Describing Templets*, as their use suggests. The tracing points may be passed through the wood obliquely, as awls; emerging just at the curved edge on the under side. The lines at 1 and 2 near the middles are to represent these tracers in position. The points should project below just enough to trace a mark and lie directly under the circular edges.

Fig. 19 shows how to use these templets to describe the curves of the teeth. When describing templet, 1, is being used on pitch line, A B, the pitch templet, A, is to be placed at A B for 1 to roll upon. Likewise, when using describing tem-

FIG. 19.



plet 2, the hollow pitch templet, B, is to be in position on A B, with A removed. Similarly for C D, and the Fig. 19 will describe the rest.

Numerous experiments may be made with such templets until familiarized. Thus it will be found that when 1 is half as large as C D, a straight line is described for the flank. Similarly a straight flank is obtained in A B, when 2 is half its size. These straight lined flanks are radial flanks.

Tooth-curves may thus be drawn on metal plates, and subsequently cut carefully to the line. Such a pattern may be used for turning up a cutter by, or for marking out the teeth on a gear pattern of wood. In the latter case, it may be called a scribe templet. And it is convenient to mount this upon a rod attached to swing around a center pin to the gear. Either of these operations involves two eye-and-hand processes, as, for instance, dressing up the scribe templets, and then turning the cutter by it. But it is not convenient to reduce the number further, though, by aid of an *odontograph*, described in Part II of this number the inconvenience of resorting to pitch and describing templets is avoided, from the fact that the odontograph serves as a ready-made scribe templet, as exact as a special scribe templet laid out and dressed up, as above described, would be, and which is also adapted to being mounted to swing around the center of the gear.

# PART II.

ON A

# NEW ODONTOGRAPH:

*An Instrument for laying out*

THE TEETH OF GEAR WHEELS.

## THE WILLIS ODONTOGRAPH.

THOSE who have used, or even studied the *Odontograph* of Professor Willis, of Cambridge, England, cannot be otherwise than forcibly impressed with the admirable simplicity and great utility of the instrument. Indeed, it has attained to such general favor that it is not only in the hands of a large proportion of the constructors of mill gearing in Europe and America, but in every work of importance on Cinematics, Principles of Mechanism, Machinery and Millwork,

&c., published since its discovery in 1838. In fact, the instrument is worthy of a place in the tool chest of every millwright and machinist; particularly those who have no. better understanding of the mode of delineating gear teeth, than to use, for generating the faces of teeth with radial flanks, describing circles which are the opposite pitch circles themselves, instead of circles one-half as large.

Although the teeth of wheels, laid out by the aid of the odontograph, are only approximately correct, such teeth are not very far from correct working forms. This fact was practically demonstrated to the writer at the justly-celebrated machine factory of Wm. Sellers & Co., of Philadelphia, where a pair of heavy cut gears were seen in action, running with extraordinary quietness. On inquiry as to the remarkable working quality of these gears, it was stated that the cutters were made expressly for them in strict conformity with the Willis odontograph; which, in the present instance, it is fair to suppose, was an experiment to test the odontograph teeth, as compared with those laid down correctly by theory. Teeth formed thus carefully, even though they be only approximate, are much to be preferred to the rattling, guess shapes; and amply repay the trouble required to secure them. It is therefore much better, in cases where the theory of gearing is not understood, to resort to an odontograph, than to jump blindly at a rattle trap.

For example: a common form among millwrights is obtained by taking the middle point of a tooth at the pitch line for a center, and describing, by arcs of circles, the nearest flanks of the next adjacent teeth, and the further faces of the same two teeth. Describing thus four arcs from the center of each tooth, all the faces and flanks are laid out. This gives flanks which are concave, with radii which are less than those of the convex faces, while the opposite relation is known to be required in correctly laid-out teeth. This rule probably gives shapes which are worse than mere guess shapes.

### A GOOD ODONTOGRAPH DESIRABLE.

The delineation of correct working gear teeth according to theory for individual cases is a very easy matter when the method is understood, but not so easy as to use a convenient odontograph; and hence, a good odontograph is advantageous to all designers of gearing.

### ESSENTIAL QUALITIES OF A GOOD ODONTOGRAPH.

An odontograph, to give the highest satisfaction, should be one which leaves nothing to be desired; neither in the degree of approximation, nor the general form of tooth it gives.

Those who have employed the odontograph of Prof. Willis, cannot fail to have noticed that in wheels of many teeth the form is very different from that obtained by laying out the teeth of the epicycloidal form with radial flanks,* which form is the most common and usually the most acceptable. Also the odontograph must be simple, and easy of application.

### PECULIARITIES OF THE WILLIS ODONTOGRAPH TOOTH.

A tooth of a wheel of eighty teeth, as laid down by aid of the Willis odontograph, is shown in Fig. 1; and also, a little to one side in dotted lines, the same tooth of epicycloidal form with radial flanks, as calculated to work with a second wheel of twelve teeth. The clearance curves are not put in. The odontograph tooth exhibits very marked peculiarities, a noticeable one being the greater general inclination of the side of the odontograph tooth to the radius of the wheel; giving rise to unnecessary obliquity of action, and consequent mutual crowding of the wheels against their shafts.

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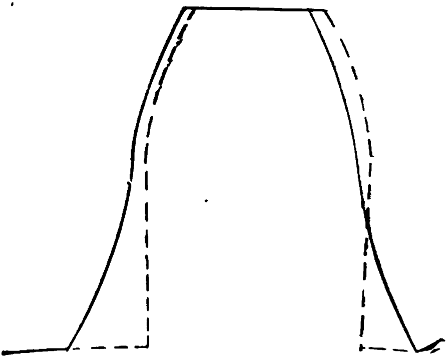
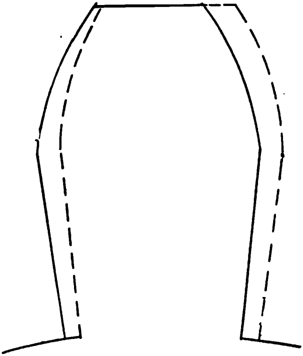
* When these teeth would appear too weak at the root of these, *clearance curves* are usually struck in, or curves between the flanks which the points of the teeth of the opposite wheel will just clear, up to which the roots of the teeth may be thickened for the purpose of greater strength.

FIG. 1.



Prof. Willis limits his odontograph to twelve teeth for the least number, and for this the teeth have radial flanks as shown in Fig. 2, which works with a wheel of eighty teeth. This limitation causes the teeth of small wheels to be narrow at the base, and of large wheels to be unnecessarily wide. In wheels having a not unusually large number of teeth, the flanks start out from the pitch line towards the center with a divergence from each other approaching 30°.* In

FIG. 2.



other words, the odontograph teeth approach the narrowness of base of epicycloidal teeth with radial flanks when the latter are objectionably narrow, as in wheels of few teeth; and much wider, indeed excessively wide, when, for the same wheels, epicycloidal teeth are of good form. Also a marked peculiarity, as exhibited in Fig. 2, is the decided angular junctions of the flanks and faces.

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* See Fairbairn, Mills and Millwork, Part II, Plate IX, Fig. 1. A. Lambert, Cinematique, Pl. 14 M. CH. Viry Cours de Méchanique, p. 194, Fig. 30.

## RESTRICTION OF THE WILLIS THEORY, AND MODIFIED FORM OF ODONTOGRAPH.

These undesirable features are, however, in the main, no fault of the general theory of Prof. Willis for the formation of teeth with circular arcs, but appear to be due to restrictions imposed, for the purpose of simplifying the tables for, and application of, the odontograph itself. The restrictions which contribute mostly to this are the fixed angle of 75°, and the constant length, relatively to the pitch, of the perpendicular to the line of action. By restricting the theory in a different manner, a certain modification of the instrument and its application may be effected, which gives straight and more nearly radial flanks for all wheels, and circle arc faces. This will be treated further on.

## NEW FORM OF ODONTOGRAPH.

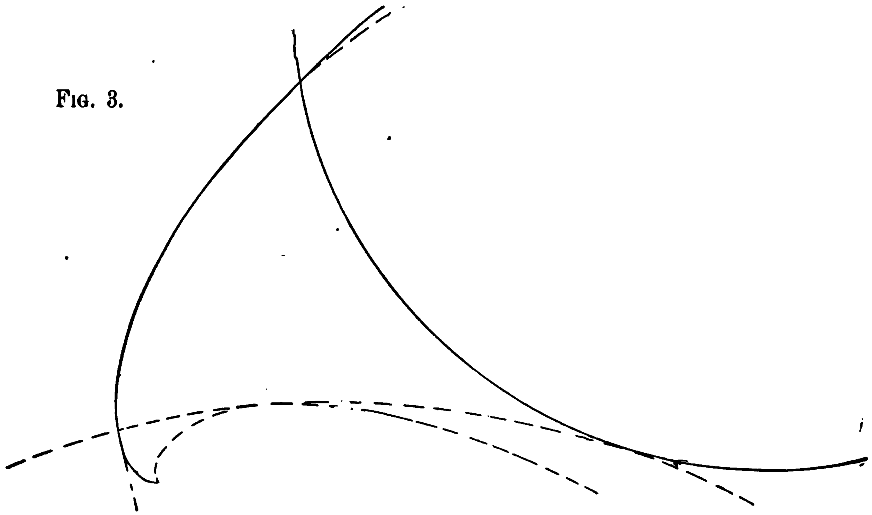
For those who will adopt nothing which is so incomplete as a circle arc for a tooth face, but will lay down the correct curves, and for even the less theoretical who would prefer to accept a good practical *short-cut* to a correct result than to go by guess, I would offer the new odontograph, which is now to be described: an instrument which in no circle, arc, mode or form, resembles any thing above referred to.

This new form of odontograph is adapted to giving curved as well as radial flanks, also faces which agree with almost undiscoverable nearness with the epicycloidal face. The face curve, as laid down by the instrument, is exactly normal to a tangent to the pitch line drawn from the middle point of a tooth, and it intersects the addendum circle precisely where the true epicycloidal curve proper to the tooth in question does.

Now, as the pitch line and its tangent pass either way from the middle of the tooth, they depart from each other slightly in distance, and more so in direction. A circle arc, approximating to a tooth face, will generally have a radius considerably greater than the half thickness of a tooth; but for a circle arc to be normal to the above-named tangent and to the pitch circle at the side of a tooth, both at the same time, its radius must be much less than the half thickness of a tooth, as is evident on inspection; and hence, the circle arc cannot osculate closely with the epicycloid, and at the same time be normal to the pitch circle, as Figs. 1 and 2 show. The most closely osculating curve must, therefore, be one which rapidly changes curvature; and such a curve will be very nearly normal to the above mentioned tangent to the pitch circle.

The odontograph should, therefore, give a curve which changes curvature rapidly. Now, as the epicycloidal curve is normal to the pitch line, and very nearly so to the tangent to the pitch circle drawn from the middle of a tooth, it is clear that if a curve of rapidly changing curvature be so placed as to be normal to the tangent, as above described, and at the same time intersecting the addendum circle at the same point that the epicycloidal curve required for the tooth considered does, it will represent the epicycloidal tooth face with great precision. This truth is rendered apparent to the eye in Fig. 3, in which the full line is the odontograph curve and the dotted line the required epicycloid. The flank is also drawn in to give an idea of the conformity of the whole tooth, as delineated by aid of the instrument, with the true epicycloidal tooth. These teeth belong to a wheel, the pairs of which are of equal size. The object of making the curve perpendicular or normal to the tangent, as above described, instead of the pitch circle, is to render the odontograph more convenient of application; and it is found that the curve selected, in the manner hereafter explained, meets the radius for extending the side of the tooth into the

Fig. 3.



flank with an error of tangency, which is too small to be readily detected.

With these explanations, it becomes evident that the leading propositions of the new odontograph are:

1st. That it gives a curve of rapidly changing curvature, having the closest possible osculation with the epicycloid, or hypocycloidal, and at the same time be of general application.

2d. That the curve for a tooth face, given by the instrument, be normal to a tangent line to the pitch circle at the middle of a tooth.

3d. That this curve intersect the addendum circle precisely where the epicycloidal curve proper to the tooth in question does.

### FORM OF CURVE ADOPTED.

The curve adopted as conforming most closely, in general, with limited initial portions of the epicycloid, is the *logarithmic spiral*, the curve being formed on the edge of the instrument itself so that it may serve as the describing templet for tooth curves.

This curve appears to possess the highest degree of adaptation because of its uniform rate of change of curvature, and also because this rate can be assumed at pleasure.* Furthermore, the analytical relation of this spiral to the problem is simple.

### FORM AND APPLICATION OF INSTRUMENT.

In planning this odontograph, the leading endeavor was to make its practical application the most convenient possible, without a sacrifice from accuracy of form of tooth obtained by it.

That the reader may learn the leading features of the instrument before proceeding to the details of its demonstration, its form and mode of application will be first explained.

The odontograph is shown full size in Fig. 4, frontispiece, as of suitable capacity for laying out all teeth below six inches pitch. Its capacity, however, is extended to any degree by simply prolonging the curved edges. It should be made of metal, because it is intended that the instrument, when desired, may be used directly for a scribe templet, in which use it will be subject to wear from the passes of the scriber. It has several holes countersunk on both sides as shown, so that it may be attached by wood screws, or by bolts expressly prepared, to any convenient wooden rod, in such a manner that when the rod swings around a center-pin to the wheel, all the faces of the teeth may be described directly from the instrument itself. The desired result is thus obtained directly without intervening center points and dividers.

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*These points follow from the fact that limited portions falling within a given angle at the pole are similar figures.

In this manner the odontograph becomes a general or ready made templet, and equally valuable for guiding the draughtsman's right-line pen, or the pattern maker's steel scriber.

To place the instrument in position for drawing a tooth face, a table is used which should accompany the instrument.

From this table a value is taken which depends upon the diameters of the pair of wheels, and the number of teeth in the wheel for which the teeth are sought. This tabular value, when multiplied by the pitch, called the setting number, or simply "setting," is to be found on the graduated edge A D B, Fig. 4, of the odontograph. This done, draw the tangent D E F to the pitch line A E at the middle point E of a tooth; and lay off the half thickness E D of tooth on either the tangent line or pitch line. Then place the graduated edge of the odontograph at D, and in such position that the setting number found as above, which is 2.5, as shown in Fig. 4, shall come precisely on the tangent line at D. Also get the curved edge, H F C, so that the curve will just lie tangent to the tangent line, as at F. Then all is ready for tracing the curve for the tooth face from the pitch line through D toward B as far as needed. By turning the instrument, which is graduated on both sides, over, and doing likewise, we get the opposite face of the same tooth. Then find the proper "settings" from the tables for the flanks, and locate the instrument as shown in Fig.A, given later, by drawing tangents to the pitch line at the sides of the tooth to set the instrument by, etc., and draw the flank curves, when the tooth becomes fully delineated, with the exception of limiting the point and root. Clearance curves may also be struck in, if desired, as in any other case, see Fig. E, given later. Of course, when the setting of the instrument is once made, it may be mounted upon a rod, Fig. D, given later, as, and for the purpose described above, for drawing all the teeth.

### FOR INVOLUTE TEETH, ETC.

The curve which this instrument gives will closely represent initial portion of the hypocycloid, cycloid and involute, as well as the epicycloid, as far as required for a tooth curve, and hence the instrument is adapted for tracing teeth of these various forms, including the rack and pinion, internal gearing and involute teeth. For the latter form of teeth the table becomes very simple.

### CONDITIONS OF TOOTH CURVES.

Before proceeding to the demonstrations, it will be necessary to understand how the various curves used for tooth outlines, are conditioned.

To generate the *epicycloidal faces for teeth with radial flanks*, the generating circles, which roll upon the pitch circles, should have diameters equal to the radii of the opposite pitch lines, as set forth in Part I of this manual.

When one of the pitch circles is infinitely enlarged, we have the case of the *rack and pinion*, the epicycloid of the former being the cycloid generated by a circle half as large as the pinion; and its flanks are parallel to each other. Also the generating circle, whose diameter equals the radius of the rack, is infinite, and hence the generating circle for the pinion teeth is a straight line. This makes the faces of pinion teeth involutes, with the pitch circle the base of the involutes.

Therefore, for the rack and pinion with radial flanks, the faces of the rack are cycloids, and of the pinion involutes.

For the case of *involute teeth* for involute gearing in general, the base circles of the involutes are usually taken a little within the pitch circles; the ratio of the base circles being equal to that of the pitch circles. The relation between a base circle and the pitch circle containing it is arbitrary, being generally assumed, in each case, according to the judgment of the designer, and any rule given as regulating this simply expresses the judgment of its author. The table for involute teeth is therefore adapted to suit any assumption in this regard. In this case, however, it should be borne in mind that the tangent line, used in setting the odontograph, should be drawn to these base circles instead of the pitch circles, because the involutes are normal to the base circles. But the half pitch should still be laid off, on the pitch lines.

The case of *internal gearing* is peculiar, in that radial flanks for the larger wheel are impossible.* That flank which approaches nearest to radial, is, therefore, adopted, and this can be readily shown to be the involute, with the pitch circle its base. Observing that the larger wheel has its flanks outside, and faces inside the pitch circle, we see that the flanks must be generated by rolling a curve on its exterior. Rolling a circle thus, we see that the epicycloid generated rises more nearly in a radial direction, as the circle is increased in size; because the latter curve reaches the greater height. Carrying this to the limit, we have the infinite generating circle or straight line, and hence the involute curve. But suppose we adopt a rolling circle of negative curvature, as in Fig. 7, Part I. Now for this mode of generation, it is easily shown that the curve generated will be an epicycloid exactly similar and equal to that obtained by rolling a circle of positive curvature, outside the pitch circle as before, the diameter of which is equal to the difference of diameters of the generating circle of negative curvature and the pitch circle.* Hence, in either case, we get similar results, and the steepest epicycloid is the above named limiting one, viz., the involute.

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* See Belanger, Cinematique, p. 121.

The flanks of the larger wheel are therefore concave involutes. Using the same generatrix for the faces of the pinion, the latter will have involute faces.

The faces of the larger wheel will be hypocycloids, because internal, and generated by a circle whose diameter equals the radius of the pinion.

In *bevel wheels*, the pitch circle should be regarded as small circles of a sphere† which has its center situated at the point of intersection of the axes, or shafts of the wheels. These pitch circles are not, in the case of bevel wheels, the proper circles to lay out the teeth upon, but should be regarded as the bases of cones whose vertices are in the axes, outside the sphere, and which, developed upon a plane surface, are the desired circular arcs on which as pitch circles to delineate the teeth. The describing of teeth on these circles, being the same as on the pitch circles of spur wheels, the odontograph applies to them in the same manner, their radii number of teeth, &c., being employed in connection with the tables, instead of those of the real pitch circles as explained in Figs. B and C.

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* Bour, Cours de Cinematique, p. 120; Note. Duhamel Elements de Calcule Infinitesimal.

† Willis, p. 146; Rankine, p. 144; Fairbairn, Mill and Millwork, part 2, p. 88; A. Lambert, Cinematique, Pl. 15.

In all these cases the same rule will apply for use of odontograph, except in involute gearing, with which we have an exception as above explained.

### PARTICULAR LOGARITHMIC SPIRAL ADOPTED.

In adopting the particular logarithmic spiral for the odontograph curve, inasmuch as this curve may have an infinite variety of obliquities, it is evident that the selection is not a matter of indifference. When the obliquity, or angle between the normal and radius-vector, is very small, the arc of this spiral changes curvature less rapidly than when the obliquity is great; when the obliquity is zero, the spiral becomes a circle; and when it is 90°, the spiral is simply a radius; neither of which approximates to the epicycloid. To find that obliquity which makes the spiral best fit the epicycloid, it will probably be most satisfactory to assume an epicycloid which represents an average of those likely to be used for both curves, and adapt the spiral to it; though any ordinary logarithmic spiral will evidently conform more closely with it than the circle. The spiral which most closely osculates the epicycloid for a pair of equal pitch circles, is therefore adopted as the average spiral, because the opposite wheel may be made larger or smaller, thus making a higher or lower epicycloid. Also, it is preferable that the spiral fit for the fewest teeth, because for this the longest portion of the epicycloid would

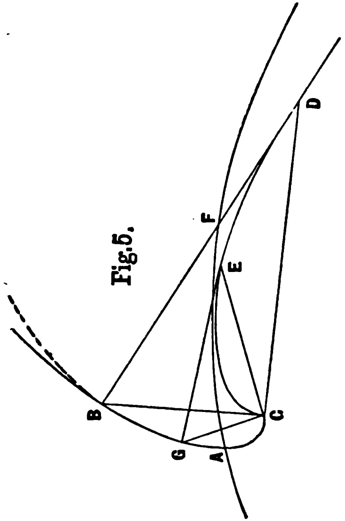

Fig.5.

be required. The fewest number assumed is ten.

Let A F, Fig. 5, represent the pitch line, A B the epicycloid above named, dotted beyond B, and the full line C A B, the osculating logarithmic spiral with its pole at C. Draw normals G E and B D. These normals will be common to both the epicycloid and spiral, if they closely osculate each other at A and B. Also E D will be the evolute of both. The point B is supposed to be 0.3 pitch* above the pitch line, as the ordinary height of tooth above pitch line. Or what is nearly the same, A B = 0.3 P. Also A G is assumed = 0.25 A B. Draw the normals or radii of curvature G E and B D. For the particular case under consideration we find for the epicycloid, DE = 2 GB. This fact is most conveniently proved graphically on account of the complication of the analysis. This solution was made with great care on a drawing, and it was also checked by an approximate analytical method. The results very nearly agreed, though as that by analysis was necessarily approximate, the graphical result was adopted. It happens to be also a very convenient value. The graphical operation was, in fact, partly by analysis thus: The normal to the epicycloid always coincides with the chord of the generating circle connecting the point of contingence of this and the pitch circle with the tracing point. This determines the direction of G E or B D, where B F, for instance, is a chord. The true radius of curvature B D, for an epicycloid, is $\frac{3}{2}$ B F* in the present case for radial flanks where the pitch circles are equal, or generating circle half as large as the directing pitch circle. This value of B D or G E, in terms of the chord, is from the well-known formula for the radius of curvature for epicycloids in general,† and hence the length is computed.

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* In all cases in this investigation, requiring dimensions to be given to the teeth, the common and well-known rule is adopted, viz. Thickness of tooth, $\frac{5}{11}$ P. Thickness of space, $\frac{6}{11}$ P. Height above pitch line, $\frac{3}{10}$ P. Depth below, $\frac{4}{10}$ P. See Willis, p. 118, &c.

Now the logarithmic spiral sought must have

$$\frac{E\,D}{B\,G} = 2 = \text{tangent of obliquity.}$$

---

* Davies and Pecks, Math. Dic., p. 222.

† See Rankine, Mach. and Millwork, p. 60, note. Redtenbacher Resultats, p. 3, rule 4. Willis, p. 142. Belanger, Cinematique, p. 103, &c.

The evolute of the logarithmic spiral, being an equal spiral,* makes C E D a spiral like C A B. Also it is well known that the radii C B and C D have B C D a right angle† when these radii vectors are drawn to the extremities of a radius of curvature B D, and similarly for E C G. This, with the second property of constant obliquity of the logarithmic spiral, makes the two triangles B C D and G C E similar. From this it follows that the triangles C B G and C D E are similar, and hence

$$CB : BG :: CD : DE,$$

or

$$\frac{DE}{BG}=\frac{CD}{BC}=\text{tangent of obliquity}=2.$$

The equation of the logarithmic spiral is

$$\log. l=a\,\theta$$

in which $a=$tangent of obliquity, or $a=2$; and therefore the logarithmic spiral sought has the equation

$$\log. l=2\,\theta$$

---

*See Courtenay's Calculus, p. 181.

† See Courtenay's Calculus, p. 173.

or

$$l=e^{2\theta}$$

or

$$\frac{l_1}{l_2}=e^{2\,(\theta_1-\theta_2)}$$

Values of the radius vector, $l$, have been carefully computed for the purpose of laying out the odontograph curve with accuracy. These were computed for the intervening angles of 22½ degrees, which divides the circumference into sixteen equal parts; and for some of the larger values the radii subdividing the angles as follows:

$$\begin{array}{llrl}
\text{For } \theta_1-\theta_2=22\tfrac{1}{2}^\circ & \frac{l_1}{l_2} & = & 2.1933 \\
\theta= \qquad \theta_1 & l_1 & = & 0.5000 \\
\text{“}=\theta_1+\ 22\tfrac{1}{2}^\circ & l_2 & = & 1.0966 \\
\text{“}=\theta_1+\ 45^\circ & l_3 & = & 2.4052 \\
\text{“}=\theta_1+\ 67\tfrac{1}{2}^\circ & l_4 & = & 5.2753 \\
\text{“}=\theta_1+\ 90^\circ & l_5 & = & 11.5700 \\
\text{“}=\theta_1+101\tfrac{1}{4}^\circ & l_{5\frac{1}{2}} & = & 17.1350 \\
\text{“}=\theta_1+112\tfrac{1}{2}^\circ & l_6 & = & 25.3700 \\
\text{“}=\theta_1+123\tfrac{3}{4}^\circ & l_{6\frac{1}{2}} & = & 37.5820 \\
\text{“}=\theta_1+135^\circ & l_7 & = & 55.6560
\end{array}$$

With these values, the odontograph, as shown in Fig. 4, was laid out, not only the curve A D B, but C F H; these being similar and equal as above explained. From the fact that tangent of obliquity $=2$, we may, when a radius $l$ is laid off for a point on A D B, draw a line at right angles, and lay off on it $2l$, which will give a point on C F H.

### EQUATION OF THE ODONTOGRAPH CURVE.

In the analytical demonstration of the applicability of this curve, an equation will be needed for it, which can be combined with those for the curves which are proposed for the outlines of teeth. For reasons which will appear hereafter, an equation is required which gives the relation between the abscissas *a e* and the ordinates *e b*, Fig. 6. Let *a c* be the radius of curvature of the spiral at *a*. Let fall upon this a perpendicular *b e*. Also prolong *a c* to *d*. Some point *d* can always be found with which, as a center, a circle may be struck through *a* and $b$. Now let us propose that $c\,d = q \times e\,b$ as a relation sufficiently exact for any possible height of tooth, $e\,b$, above pitch line.

Then we will have from the property of the circle,

$$a\,e\,(2\,a\,d - a\,e) = \overline{e\,b}^2,$$

or calling $a\,e = y$ and $e\,b = x$, $a\,c = \tau$, we have

$$y\,[2(\tau + q\,x) - y] = x^2$$

whence

$$\tau = \frac{x}{2}\left\{\frac{x}{y} + \frac{y}{x} - 2q\right\} \quad . \quad . \quad (1)$$

If in this equation, $q$ proves to be constant, we have the equation desired. To this end the following results have been very carefully determined graphically, this method being the only one admissible for complexity of formulas:

For $\tau = 2.0$ inch $x = 1.1$ inch $q = .596$
$\tau = 2.0$ inch $x = 2.0$ inch $q = .588$
$\tau = 4.4$ inch $x = 1.0$ inch $q = .561$
$\tau = 4.4$ inch $x = 2.5$ inch $q = .557$
$\tau = 4.4$ inch $x = 4.5$ inch $q = .562$

For $\tau=9.0$ inch $x=0.5$ inch $q=.595$
$\tau=9.0$ inch $x=9.0$ inch $q=.599$
$\tau=9.5$ inch $x=2.2$ inch $q=.555$
$\tau=9.5$ inch $x=5.5$ inch $q=.600$

Mean . . $q=.579$

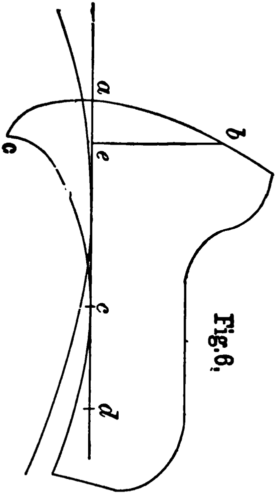

Fig. 6.

These figures show that $q$ is very nearly constant; in fact, the differences can only be accounted for as errors of the graphical operation; errors which might be expected for the reason that the point *d*, Fig. 6, may vary its position considerably, and yet cause but slight variations at *b*. We may therefore adopt this, not only as constant, but as the correct value of $q$ for the particular spiral chosen, and this value was employed in the formulas when computing the tables for the odontograph. Equation (1) is entirely empirical, and will not apply to the spiral for a very great range; though to an extent greater than ever needed for the present purpose, it is practically exact. It is necessary to resort to the empirical method, because the equation of this spiral cannot be expressed in rectangular co-ordinates.

## DEMONSTRATION FOR EPICYCLOIDAL SPUR GEARING.

To show that the odontograph, as above proposed, is applicable to the case of ordinary epicycloidal gearing-teeth, it will be necessary to prove that the value of $r$ to be found on the scale by which the instrument is set, can be computed when certain functional parts of the gears in question are known; and also, to show that it can be expressed in so few terms, that values of it may be computed beforehand, and arranged in tabular form, it will be necessary to resort to mathematical expressions for $\tau$, by which it can be discussed, and by which we may determine the number of quantities required to express it. For instance, if it can be expressed in terms of two different quantities, only, one table will suffice for all values of $\tau$. But if it requires three quantities for its expression, a series of tables will be necessary.

By aid of the equations of the epicycloid the above co-ordinates $x$ and $y$ may be expressed in quantities belonging to the wheels.

Let $R$ = the radius of the pitch circle of the wheel for which the teeth are sought, whether it be the larger or smaller.

Let $N$ = the number of its teeth.

Let $r$ = the diameter of the rolling

circle for generating the faces sought.

Let $n = \frac{2\pi r}{P}$.

Let P=the pitch of the teeth$=\frac{2\pi R}{N}$ $=\frac{2\pi r}{n}$.

Let $r$=the radius of curvature of the curve of odontograph, as above, the numerical value of which is also the number to be used for setting the instrument, also the tabular numbers.

Let $x$ and $y$=co-ordinates as in equation (1) of odontograph curve above, $x=0.3$ P.

Let $\theta$=an angle, which may be considered as an auxiliary quantity, simply; or as the angle which locates the center of the rolling circle.

Then the well-known* polar equation of the epicycloid is—

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* See Price's Calculus, vol. i, p. 349.

$$(\text{rad vector})^2 = (\mathrm{R}+x)^2 = \mathrm{R}^2 + r\left\{\mathrm{R}+\frac{r}{2}\right\}\left\{1-\cos.\frac{2\,\mathrm{R}}{r}\theta\right\}$$

whence

$$\cos.\frac{2\,\mathrm{R}}{r}\theta = 1 - \frac{\left\{1+\frac{x}{\mathrm{R}}\right\}^2 - 1}{\frac{r}{\mathrm{R}}\left\{1+\frac{r}{2\,\mathrm{R}}\right\}}$$

or

$$\cos.\frac{2\,\mathrm{N}}{n}\theta = 1 - \frac{\left\{1+\frac{0.3\,\mathrm{P}}{\mathrm{R}}\right\}^2 - 1}{\frac{n}{\mathrm{N}}\left\{1+\frac{n}{2\,\mathrm{N}}\right\}} \qquad (2)$$

This value of $\theta$ is in terms of the two quantities $\frac{\mathrm{P}}{\mathrm{R}}$ and $\frac{n}{\mathrm{N}}$ only, as desired.

The polar equation is here employed instead of that referred to rectangular co-ordinates, because the former is resolvable with respect to $\theta$, and the latter not. The latter, however, would have been preferable to the extent of about the difference between the height of *b*, Fig. 6, above the tangent and above the pitch line, because *b e*, which is about midway between $a$, and the middle of the tooth, is almost exactly equal to $x$ as given by the rectangular equation. But this difference is very small in wheels of the fewest teeth, say 8 or 10, and inappreciable in larger wheels.

An expression for $y$ is most readily obtained by employing the well-known* equation of $y$ in terms of $\theta$ for rectangular co-ordinates to the epicycloid.

In Fig. 7, let A B represent the face of the tooth for the wheel whose center is O. Take C D the height of the tooth above the pitch line reckoned on the center line of tooth, and=B F=0.3 P, nearly,$=x$. Let C G represent the tangent to the pitch line at the center point C of tooth, as above described.

Now, since the co-ordinates $x$ and $y$ in equation (1) of the odontograph curve represent the height of B above the tangent C G, and the distance from the face along the tangent C G to the perpendicular let fall from B upon C G respectively, or equal to B F and A $a$ respectively, very nearly, we have to find, by help of

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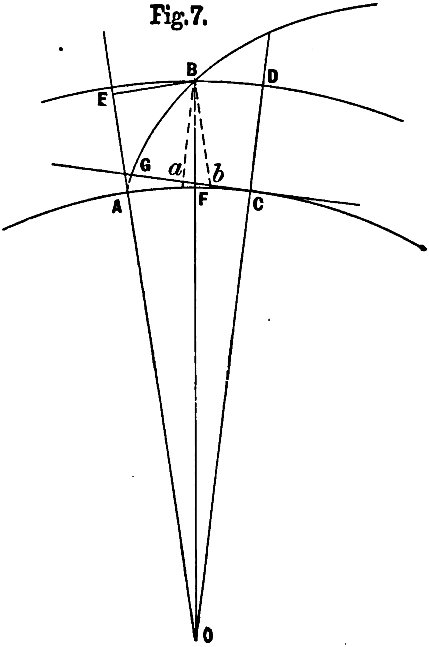
* See Price's Calculus, vol. i, p. 283.

Fig. 7.



the equation of the epicycloid, between $y$ and $\theta$, the value of A $a$. But this equation gives E B, or A *b*. Drawing B *b* parallel to A E, and B *a* parallel to D C, we have

$$ab : \mathrm{FB} :: \mathrm{AC} : \mathrm{CO}$$

or

$$ab : \; x \; :: \tfrac{5}{22}\,\mathrm{P} : \mathrm{R}$$

since the thickness of tooth $= \frac{5}{11}$ P. Hence

$$ab = \frac{5}{22}\,\frac{x\,\mathrm{P}}{\mathrm{R}} = \frac{5}{22}\,\frac{0.3\,\mathrm{P}^2}{\mathrm{R}} = \frac{\mathrm{P}^2}{\mathrm{R}}0.06819$$

and hence, by aid of the equation of the epicycloid,

$$\mathrm{A}\,a = y = \mathrm{E\,B} - a\,b = \left(\mathrm{R} + \frac{r}{2}\right)\sin.\,\theta$$
$$- \frac{r}{2}\sin.\left(\frac{2\,\mathrm{N}}{n} + 1\right)\theta - a\,b$$

or

$$\frac{y}{\mathrm{P}} = \frac{\mathrm{R}}{2\,\mathrm{P}}\frac{n}{\mathrm{N}}\Big\{\frac{2\,\mathrm{N}}{n} + 1\Big)\sin.\,\theta$$
$$-\sin.\left(\frac{2\,\mathrm{N}}{n} + 1\right)\theta\;\Big\} - \frac{\mathrm{P}}{\mathrm{R}}0.06819 \qquad (3)$$

This equation, together with (2), enables us to compute $\frac{y}{\mathrm{P}}$ when $\frac{\mathrm{P}}{\mathrm{R}}$ and $\frac{n}{\mathrm{N}}$ only, are known, $\theta$ being eliminated between (2) and (3).

Observing that $x=0.3$ P, equation (1) may be written

$$\frac{\tau}{\text{P}}=0.15\left(\frac{.3\ \text{P}}{y}+\frac{y}{0.3\text{P}}-2q\right) \quad . \quad (4)$$

So that $\frac{\tau}{\text{P}}$ becomes fully known when the two quantities $\frac{\text{P}}{\text{R}}$ and $\frac{n}{\text{N}}$ belonging to the pair of wheels under consideration become known. Or, observing that $\frac{\text{P}}{\text{R}}=\frac{2\,\pi}{\text{N}}$, the most convenient forms of the equations for computation are

$$\cos.\frac{2\ \text{N}}{n}\theta=1-\frac{\left(1+\frac{1.885}{\text{N}}\right)^2-1}{\frac{n}{\text{N}}\left(1+\frac{n}{2\ \text{N}}\right)} \quad . \quad (5)$$

$$\frac{y}{\text{P}}=0.07958\ \text{N}\ \frac{n}{\text{N}}\left\{\left(\frac{2\ \text{N}}{n}+1\right)\sin.\theta\right.$$
$$\left.-\sin.\left(\frac{2\ \text{N}}{n}+1\right)\theta\ \right\}-\frac{0.42841}{\text{N}} \quad . \quad (6)$$

$$\frac{\tau}{\text{P}}=0.15\left(\frac{0.3\ \text{P}}{y}+\frac{y}{0.3\ \text{P}}-1.158\right) \quad . \quad (7)$$

These formulas enable us to compute $\frac{r}{P}$ when $\frac{N}{n}$ and N are known. Table I contains most of the values $\frac{r}{P}$ needed in practice, and obtained from these formulas. The tabular value of $\frac{r}{P}$ when multiplied by P gives $r$ the index number for setting the odontograph. Examples will be given hereafter, illustrating the use of the instrument and table in delineating gear teeth.

### DEMONSTRATION FOR THE RACK AND PINION.

For this case formula (6) becomes indeterminate, and hence fails. It will, therefore, be necessary to deduce others.

The cycloid for the tooth faces on the rack, flanks radial, will have the following values of $x$ and $y$* ;

$$x=\frac{r}{2}\left(1-\cos.\ \theta\right)$$

---

* Price's Calculus, vol. i, p. 280.

$$y=\frac{r}{2}\left(\theta-\sin.\theta\right)$$

and hence

$$\cos.\theta=1-\frac{1.2\,\pi}{n}=1-\frac{3.76992}{n}\quad . \quad . \quad (8)$$

$$\frac{y}{\mathrm{P}}=\frac{n}{4\,\pi}\left(\theta-\sin.\theta\right)\quad . \quad (9)$$

By expanding cos. $\theta$ into a series and dropping all after $\theta^2$, by which we involve an error of only $\frac{1}{28}$ part for the case of 8 teeth, and much less for a greater number; and similarly expanding sin. $\theta$ into a series, omitting the terms following $\theta^3$ with an opposite error of $\frac{1}{50}$ for 8 teeth, giving a residual error of about $\frac{1}{50}$ we may write,

$$\frac{y}{\mathrm{P}}=\frac{1}{10}\sqrt{\frac{2.4\,\pi}{n}}\quad . \quad . \quad (10)$$

This, combined with (7), was used for computing the tabular values for racks given at bottom of Table I.

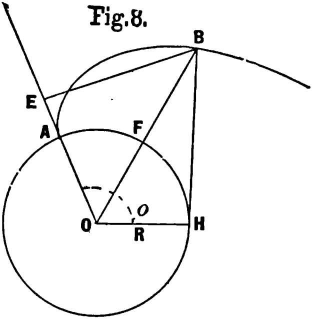
For the involute faces of the pinion we readily obtain special formulas.

Let A H, Fig. 8, be regarded as the pitch line, A B as a tooth face of the involute form, and B F height of tooth above the pitch line and equal to $0.3\ P = x$. We will have $a\,b$ same as before.

$$\therefore\ y = \mathrm{E\,B} - a\,b$$

as in Fig. 8.

Fig. 8.



$$\mathrm{E\,B} = (\mathrm{R} + \mathrm{B\,F})\ \text{arc}\ \mathrm{B\,O\,E}\ \text{nearly}$$
$$= (\mathrm{R} + 0.3\ \mathrm{P})\ (\theta - \mathrm{B\,O\,H})$$
$$\mathrm{H\,B} = \mathrm{A\,F\,H} = \mathrm{R}\,\theta = \mathrm{R}\ \text{tang.}\ \mathrm{B\,O\,H}.$$

Hence

$$y = \mathrm{E\,B} - a\,b = (\mathrm{R} + 0.3\ \mathrm{P})\ (\theta - \text{tang.}^{-1}\,\theta) - a\,b$$

or

$$\frac{y}{\mathrm{P}} = \left(\frac{\mathrm{N}}{2\,\pi} + 0.3\right)\left(\theta - \text{tang.}^{-1}\,\theta\right) - \frac{0.42841}{\mathrm{N}}\quad .\quad (11)$$

and

$$\overline{BH}^2 = \overline{R\theta}^2 = (R + 0.3\ P)^2 - R^2$$

or

$$\theta^2 = \left(1 + \frac{0.6\ \pi}{N}\right)^2 - 1 \quad . \quad (12)$$

which equations, combined with (7), were used in bringing out the tabular quantities marked "Pinion" at the bottom of Table I.

It is observed that we have one line of values only for each, the rack and pinion, the reason for this is that the value of $\frac{\tau}{P}$ for the rack and pinion become known in terms of one arbitrary quantity instead of two.

### DEMONSTRATION FOR WHEELS WITH INVOLUTE TEETH.

Having a set of values of $\frac{\tau}{P}$ for involute faces, we are at once prepared to lay out any involute gear-teeth with the odontograph, from the fact that in this form of teeth each wheel is independent of all others. In this case, however, we must observe that the tangent line E D, Fig. 4, should be drawn to the base circle instead of from the pitch circle, as stated above, under *conditions* for involute teeth. The half thickness of tooth should still be laid off on the pitch line because this is the proper line to reckon tooth thicknesses on. The tabular values are therefore the same as those for *Pinion*, Table I.

### DEMONSTRATION FOR WHEELS IN INTERNAL GEAR.

First consider the larger wheel, in which the faces are internal and hence hypocycloids. Equations (2) and (3) apply to this case by changing the signs of $r$ and $x$, and hence

$$\cos. \frac{2\,R}{r}\,\theta = 1 - \frac{1 - \left(1 - \frac{x}{R}\right)^2}{\frac{r}{R}\left(1 - \frac{r}{R\,2}\right)}$$

or

$$\cos. \frac{2\,\mathrm{N}}{n}\theta = 1 - \frac{1 - \left(1 - \frac{0.6\,\pi}{\mathrm{N}}\right)^2}{\frac{n}{\mathrm{N}}\left(1 - \frac{n}{2\,\mathrm{N}}\right)} \quad (13)$$

and

$$\frac{y}{\mathrm{P}} = \frac{\mathrm{R}\;n}{2\,\mathrm{PN}} \left\{ \left(\frac{2\,\mathrm{N}}{n} - 1\right) \sin.\,\theta - \sin.\left(\frac{2\,\mathrm{N}}{n} - 1\right)\theta \right\} + \frac{0.6\,\pi}{\mathrm{N}} 0.2273 \quad (14)$$

which, combined with (7), were used in producing the second part of Table I. The flanks of the pinion are supposed to be radial, and those of the wheel involutes, as set forth under *conditions* for these wheels. The external involute flanks of the wheel are laid out by aid of the value given in Table I for "*pinion,*" the tangent line E D, Fig. 4, being drawn to the pitch line from the middle of a *space*, to suit the present reversed condition from outside to inside.

The faces of the pinion, being also *involutes*, the same set of tabular values last named apply.

In this case of internal gearing, it should be observed that the sum of the diameters of the two describing circles should never exceed the difference of the diameters of the pitch lines of the annular and internal wheels, for avoiding positive interference of the teeth.—MacCord.

### CASE OF BEVEL WHEELS.

Observing the explanations, page 71, and for Figs. B and C, regarding bevel wheels, either epicycloidal or involute teeth may be laid out for these wheels.

### RULES AND EXAMPLES FOR FINDING THE TABULAR VALUES FROM THE TABLES.

To show the application of the tables, the following rules and examples are given: Let the pair of wheels be the same for each example, the teeth simply being different; let the number of teeth in the smaller wheel be 16 and in the larger 56. Also take the same pitch for all, and equal to two inches.

The "wheel sought" means the wheel for which the teeth are sought, or the wheel which for the time being is being drawn. When the first column RATIO and teeth numbers fall between those given in the table, the tabular values are found by interpolating as seen in the following examples:

1st. *Flanks Radial.*—For faces of wheel sought, divide radius of that wheel by radius of the other wheel; or, teeth in that wheel by teeth in the other wheel of the pair. This is the first column *Ratio*, and opposite this, under proper number of teeth, find tabular value for faces.

By this rule the required quantities for hunting the setting by, for the larger wheel, are for

Ratio$=3\frac{1}{2}$. N$=56$.

and for these values, running down the column headed $\frac{\text{"R"}}{r}$ we find $3\frac{1}{2}$ comes midway between 3 and 4; and opposite them, under "56," corresponding to N, the numbers .473 and .409. Taking the mean for the interpo-

## TABLE I.

## TO LAY OUT TEETH OF EPICY

TABLE ; FOR SETTING

The Tabular values must always be multiplied by the
used in placing

| Radius of wheel sought Divided by Diameter of generating circle, or RATIOS. | | Number of Teeth in Wheel Sought ; or, | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | 8 | 12 | 16 | 20 | 30 | 40 | 50 |
| | | FOR FACES ; Flanks Draw setting tangent at middle of | | | | | | |
| Multiply by degree of Flank Curve before looking here. | $\frac{1}{12}$ = 083 | .32 | 39 | .46 | .51 | | | |
| | $\frac{1}{4}$ = 250 | .31 | .37 | .44 | .49 | .61 | .70 | .78 |
| | $\frac{1}{2}$ = 500 | .28 | .34 | .41 | .46 | .57 | .66 | .73 |
| | $\frac{2}{3}$ = .667 | .27 | .32 | .38 | .43 | .54 | .62 | .70 |
| | 1 .... | .23 | .28 | .34 | .39 | .49 | .58 | .65 |
| | $\frac{3}{2}$ = 1.50 | .19 | .25 | .29 | .34 | .44 | .51 | .58 |
| | 2 .... | .17 | .22 | .26 | .30 | .38 | .46 | .53 |
| | 3 .... | | .16 | .19 | .23 | .31 | .38 | .44 |
| | 4 .... | | .14 | .17 | .20 | .26 | .33 | .38 |
| | 6 .... | | | | | .22 | .26 | .30 |
| | 12 .... | | | | | | .20 | .23 |
| | 24 .... | | | | | | | |
| | | FOR FLANKS, Draw Setting Tangent at side of Tooth. of Internal and Flanks | | | | | | |
| Degree of Flank Curve. | 1.5 slight | .77 | .98 | 1.18 | 1.36 | 1.75 | 2.05 | 2.31 |
| | 2 good | .44 | .54 | .63 | .72 | .92 | 1.09 | 1.24 |
| | 3 more | .20 | .28 | .35 | .40 | .54 | .65 | .76 |
| | 4 much | | .16 | .23 | .25 | .34 | .42 | .51 |
| | 6 .... | | | .16 | .17 | .26 | .32 | .38 |
| | 12 .... | | | | | .19 | 24 | .28 |
| | 24 .... | | | | | | | |

For Faces of Racks, and of Pinions for Racks and Int'l
Draw Setting Tangent at Middle of Tooth regarding
number of

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Rack. | .34 | .43 | .51 | .59 | .75 | .88 | 1.01 |
| Pinion. | .32 | .38 | .44 | .50 | .62 | .72 | .80 |

## Table I.

## CLOIDAL AND INVOLUTE GEARING.

### THE ODONTOGRAPH.

Pitch in inches to obtain the setting numbers to be Odontograph.

Wheel for which Teeth are Sought.

Radial or Curved.
Tooth.—Epicycloidal Spur or Bevel Gearing.

| 60 | 70 | 80 | 90 | 100 | 120 | 150 | 200 | 300 | 500 |
|---|---|---|---|---|---|---|---|---|---|
| .85 | .92 | .99 | 1.05 | 1.11 | 1.22 | 1.36 | 1.55 | 1.94 | 2.54 |
| .80 | .87 | .93 | 1.00 | 1.06 | 1.15 | 1.29 | 1.50 | 1.86 | 2.41 |
| .77 | .83 | .89 | .95 | 1.01 | 1.11 | 1.24 | 1.45 | 1.79 | 2.32 |
| .72 | .78 | .83 | .89 | .94 | 1.03 | 1.15 | 1.36 | 1.65 | 2.10 |
| .64 | .69 | .74 | .79 | .84 | .93 | 1.05 | 1.25 | 1.53 | 1.94 |
| .59 | .63 | .68 | .72 | .76 | .84 | .95 | 1.13 | 1.40 | 1.81 |
| .49 | .53 | .57 | .60 | .63 | .71 | .82 | .97 | 1.23 | 1.60 |
| .42 | .46 | .49 | .53 | .56 | .63 | .73 | .87 | 1.08 | 1.42 |
| .34 | .37 | .41 | .44 | .47 | .53 | .61 | .71 | .90 | 1.20 |
| .25 | .28 | .30 | .32 | .34 | 37 | .42 | .49 | .60 | .82 |
|  |  |  | .19 | .21 | .23 | .26 | .31 | .40 | .57 |

when Curved.
—Epicycloidal Spur and Bevel Gearing. Faces
of Pinion Teeth.

| 60 | 70 | 80 | 90 | 100 | 120 | 150 | 200 | 300 | 500 |
|---|---|---|---|---|---|---|---|---|---|
| 2.56 | 2.75 | 2.92 | 3.08 | 3.24 | 3.52 | 3.87 | 4.51 | 5.50 | 7.20 |
| 1.38 | 1.49 | 1.59 | 1.69 | 1.79 | 1.98 | 2.23 | 2.67 | 3.22 | 4.50 |
| .86 | .95 | 1.02 | 1.10 | 1.18 | 1.31 | 1.46 | 1.67 | 2.08 | 2.76 |
| .59 | .66 | .71 | .77 | .82 | .92 | 1.06 | 1.25 | 1.64 | 2.15 |
| .43 | .48 | .52 | .56 | .60 | .66 | .76 | .93 | 1.20 | 1.54 |
| .31 | .34 | .36 | .38 | .40 | .45 | .52 | .63 | .80 | .98 |
|  |  |  |  | .22 | .25 | .28 | .33 | .47 | .60 |

Gears; for Flanks of Int'l and Sides of Involute Teeth.
space as Tooth in Internal Teeth. For Rack use
Teeth in Pinion.

| 60 | 70 | 80 | 90 | 100 | 120 | 150 | 200 | 300 | 500 |
|---|---|---|---|---|---|---|---|---|---|
| 1.12 | 1.21 | 1.30 | 1.40 | 1.48 | 1.65 | 1.85 | 2.15 | 2.65 | 3.50 |
| .87 | .93 | .99 | 1.03 | 1.08 | 1.16 | 1.27 | 1.49 | 1.86 | 2 44 |

lated value, we have .44 as the multiplier for the pitch. Hence

$$.44 \times 2 \text{ inches pitch} = .88 \text{ inches},$$

which is the index number, or the number to look out on the scale A D B, Fig. 4, which point must be brought to coïncide with the tangent D E. Then, placing the curve C F tangent to the tangent line as shown in the figure, the tooth face curve may be drawn.

For the smaller wheel we have

$$\text{Ratio} = .286+, \text{ and } n = 16.$$

Running down the column headed *Ratio* we find no ratio = .286, but .250 and .333. Interpolating by the well-known method of proportional parts, we find, in column headed "16," for the multiplier the number .43. Hence

$$.43 \times 2 = .86 \text{ inches}.$$

which is the setting or index number required for the small wheel.

2d. *Flanks Curved.*—For flanks of wheel sought, assume degree of flank curve or first column *Ratio*, as 1. 5, 2, etc., and find tabular value opposite, under proper number of teeth. Then for faces of wheel sought, divide radius of that wheel by radius of the other wheel, and multiply by the degree of flank curve adopted for the other wheel. This is the first column *Ratio*, opposite which, under "Faces," find tabular value.

Under this rule the flanks can be curved to any extent desired, depending on the "degree" of flank curve assumed Assume that for the larger wheel 3 degree, and for the small wheel 2 degree. Then for the flanks, we have from the table opposite 3 and 2 respectively in the first column, the tabular values for the

Large wheel, 3 degree, tab. val. .82
Small " 2 " " " .63

which for the 2 inches pitch are to be multiplied by 2.

Next, for the faces of the large wheel the ratio of radii is 3.5 which, multiplied by the degree of flank curve, gives 3.5×3=10.5 to look for in the first column of table, and opposite which, under 56 teeth, we find the tabular value required, which, by interpolating, is found to be .28, and this multiplied by the Pitch=.28×2=.56.

Also for the faces of the small wheel the ratio of radii is 0.29, and this multiplied by the degree of flank curve is .29 ×2=.58, to be found in the first column of table under faces, and under 16 teeth, which gives the tabular value .39, and this multiplied by the pitch gives .39×2=.78 for the setting.

3d. *Interchangeable "Sets" of Wheels.*—Divide radius of each wheel by some one number not greater than the radius of smallest wheel. This gives the first column *Ratio*, for face and flank, both the same for one wheel, and opposite which, under "Face" or "Flank" respectively, find tabular value. Or if preferred, divide teeth numbers instead of radii by a number not greater than the number of teeth in the smallest wheel.

Under this rule the number, by which to divide the tooth numbers to obtain the first column ratio, should be chosen with a view to what number of teeth shall be in the smallest wheel of the contemplated set, and not exceeding that smallest number of teeth. For teeth as laid out by the Willis odontograph, that number would be 12, for which the wheel of 12 teeth would have radial flanks. For the present example take the number 8, for which the wheel of 8 teeth will have radial flanks.

Dividing the tooth number by 8 we obtain the values

$$\frac{56}{8}=7 \text{ for large wheel,}$$

and

$$\frac{16}{8}=2 \text{ for small wheel,}$$

for the values to find in the first column of table.

Hence for the large wheel we find the tabular values for

Face .32 and Flank .40

and for the small wheel we find opposite 2 the values for

Face .26 and Flank .63,

which values multiplied by the 2 inches pitch give the setting values.

4th. *Rack and Pinion.—For Radial Flanks* use for faces the two lower lines of table. *For Curved Flanks* find tabular value for pinion faces in lowest line. For flanks of pinion choose degree of curve and find tabular value under "Flanks," as for other wheels. For faces of rack divide number of teeth in pinion by degree of curve, which take for number of teeth in looking opposite "Rack." Flanks of rack are still parallel, but may be arbitrarily curved beyond half way below pitch line.

First, for radial flanks we find the tabular values in the lowest 2 lines of the table, for which we have for pinion faces .44 and rack faces .51, and the flanks are radial for pinion and parallel for rack.

Second, for flanks curved 2 degree (flanks of pinion only), we find the tabular values to be rack faces .34 with flanks parallel; and for the pinion flanks 2 degree, the tabular value is .63 for flanks with faces as before.

5th. *Internal Gears.*—For tooth curves within the pitch lines, divide radius of each wheel by any number not greater than radius of pinion, and look in the table under "Flanks." For curves outside pitch line use lower line of table; or, divide radii by any number and look under "Faces." In applying Inst. draw tangents at middle and side of *space*, for internal teeth, or divide the tooth numbers instead of the radii if preferred; for the curves inside the pitch line divide the numbers by 8, and we obtain 7 degree and 2 degree respectively for wheel and pinion, giving the tabular values .31 and .63 respectively.

For the curves outside the pitch lines, the lower line of the table gives .84 and .44 respectively.

6th. *Involute Teeth.*—For tabular values look opposite pinion, under proper number of teeth, for each wheel. Draw setting tangent from "base circle" of involute, at middle of tooth. For this the instrument gives the whole side of tooth at once.

For this case we find the tabular values .84 and .44.

All the above examples of tabular values are collected in the following table for convenient reference as examples to aid the beginner in working out "settings."

These examples show that the instrument, and the above Table I, meets all cases of ordinary gearing.

But it is a great convenience to have tables for all the usual forms of gearing; one table for each case given separately from the rest. Such multiplying of tables would be unsuited to the purposes of this elementary treatise. A series of six tables for as many styles of gearing, including all practical cases, is given in a manual, a copy of which accompanies every instrument sent out, also a great variety of rules for practice.

## EXAMPLES OF TABULAR VALUES AND SETTING NUMBERS.

For a pair of 16 and 56 teeth; radii 5.09 and 17.82 inches respectively; and 2 inches pitch.

| | Kind of Gearing. | Number of Teeth | Kind of Flank. | Ratio Radii | 1st Column Ratio. | | Tab. Val. | |
|---|---|---|---|---|---|---|---|---|
| | | | | | Flank | Face | Fl'k | F'ce |
| 1 | Epicycloidal, | Small | Radial | .29 | Radial | .29 | — | .44 |
| | Radial Flanks. | Large | " | 3.5 | " | 3.5 | — | .44 |
| 2 | Epicycloidal, | Small | Curved 2 deg | .29 | 2 | .87 | .63 | .39 |
| | Curved Flanks. | Large | " 3 " | 3.5 | 3 | 7. | .82 | .28 |
| 3 | Epicycloidal, | Small | "Sets," Divide | 2. | 2 | 2. | .63 | .26 |
| | Interchang'ble. | Large | Radii by 2.55 | 7. | 7 | 7. | .40 | .32 |
| 4 | Epicycloidal, | Pinion | Curved 2 deg | | 2 | Pinion | .63 | .44 |
| | Internal. | Wheel | Int face 7 deg | 3.5 | Pinion | 7.* | .84 | .31 |
| 5 | Epicycloidal, | Pinion | Curved 2 deg | | 2 | Pinion | .63 | .44 |
| | Rack & Pinion. | Rack | Parallel | | Parallel | Rack | — | .34 |
| 6 | Involute | Small | Face & Flank | | Pinion | | .44 | |
| | Gearing. | Large | One Curve | | Pinion | | .84 | |

The Tabular values multiplied by Pitch gives the Setting numbers.

* The face being here internal, the tabular value is to be found under "Flanks."

If Bevels, use ratio radii .082 and 12.25; and teeth numbers 16.6 and 208.8 respectively.

### FOR LOCATING THE ODONTOGRAPH.*

After the "*settings*" have been made out from the above Table, the odontograph may be laid down in position, and the tooth curves traced as follows:

1. Draw a circle for the pitch line of the "*wheel being drawn or wheel sought*," as shown in Fig. A.

2. Find the center points, C, for all the teeth. This must be done accurately, as the "spacing" is as important in correct working and quiet gears, as the profiles of the teeth.

3. Lay off the half-tooth thickness, C D and C E, for all the teeth, making proper allowance for clearance.

4. Draw the setting Tangents, or lines A C E and B D, the former perpendicular to the radius, C F, and the latter perpendicular to the radius, D G.

5. For the *face*, set the odontograph on the line A E, in such a way that the "setting" for the *face* falls on the line A E. at H, and the hollow edge of the

---

* Copyright in 1882, by S. W. Robinson as far as to include Fig. G.

instrument just comes tangent to or even with the line A E. Then all is ready for drawing the Face curve, the odontograph being the tooth templet.

6. For the *Flank*, set on the line, B D, so that the "setting" for the Flank falls on the line, B D, at D; and with the hollow edge of the instrument just tangent to the line at B. Then all is ready for drawing the Flank curve.

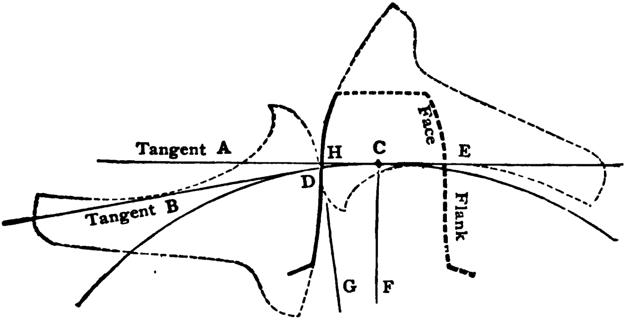


FIG. A.—How to place the Odontograph.

The odontograph is thus a curved ruler for use in drawing the tooth curves.

7. For the *Faces*, the point of the in-

strument should be turned toward the center of the wheel; and for the *Flanks*, from it as shown. For exception in Annular wheels, see rule 5th.

### FOR BEVEL WHEELS.

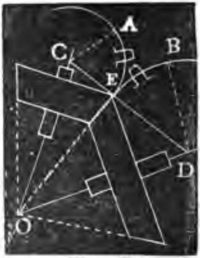


FIG. B.
**Draw'g for Bevel Wheels.**

Fig. B shows how a pair of friction wheels should be constructed; that is, wheels without teeth. The rolling surfaces of such wheels should both converge at a common point, with the intersection O of the axes. Such surfaces become what are called "pitch *surfaces*" or pitch cones, for Bevel Gears, because they cut through the teeth at the height of the pitch lines. In fact, the limits or edges of these surfaces, for a wheel, are the outer and inner pitch lines themselves. The outer pitch lines, for the two wheels, touch each other at E in Fig. B. The *Faces* of teeth or cogs, of course, are above, and the *Flanks* below these lines.

That the outer ends of teeth of both wheels shall lie even with each other at E, while running together they must be formed to some other surface than those of the planes of the pitch lines. Strictly, they should be spherical surfaces, with center at O.

But for practical convenience and sufficient accuracy, they are conical, the tips of which cones are at C and D. The line C D must be perpendicular to E O. On these cones the teeth should be laid out. But these cones may be rolled out flat, as shown in Fig. B, a part of one of which is C E A, and of the other D E B. They are simply circles with centers at C and D respectively. On these circles draw the teeth as in spur wheels. A pair of teeth is shown thus drawn.

### TOOTH, ARM AND HUB.

The bevel wheel D, Fig. B, is shown more fully and definitely drawn in Fig. C, where the letters B, D, E, O stand for the same as in Fig. B. All the surface lines of the teeth should run to O, as shown for the tooth b i v d. The pitch surface runs through E c; the outer and inner pitch circles being shown in the lines E o, and c p, in this side view. The rim crown or web of the wheel, on which the teeth stand is i l q v. E D and d q are perpendicular to E O. The tooth T is to be laid out on the circle E B, struck from the center D, in the same manner as explained in Fig. A.

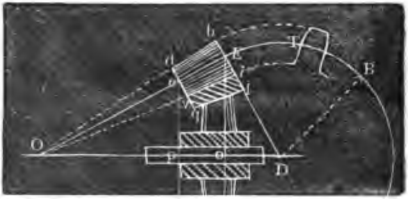


Fig. C.

Hence the simple *Rule* for Bevel Wheels, viz: Draw the wheels in the manner shown in Fig. B, and also the circles A and B on which to lay out the teeth as though C and D were centers of spur wheels. In working out the settings for this, go by the number of teeth in the full circles of A and B, to the centers C and D, just as though these circles were full of teeth to the same pitch as that of the actual wheels. These are the tooth numbers under which to look, in the tables, and also from which to work out the 1st column quotients.

The 1st column quotients of *Tables* may also be obtained by dividing the length C D, by A C or by E D, as the case may be.

For shafts at right angles, we may proceed thus, and avoid the drawing.

For C O D=90°, *Rule* for finding the number of teeth to work out the setting by.—Divide the number of teeth in the actual wheel being drawn by the number in its mate; square this quotient and add 1. Then extract the square root and multiply by the number of teeth in the wheel being drawn. This gives the number of teeth for the circle A or B, as the case may be.

### EXAMPLE OF BEVEL WHEELS WITH SHAFTS AT RIGHT ANGLES. RULE 2.

| | | Pinion. | Wheel. |
|---|---|---|---|
| Actual number of teeth in wheels | | 16. | 56 |
| Tooth numbers for A and B, . | | 16.6 | 204. |
| 1st. column quotient, 2 degree . | | 12.2 | .081 |
| Settings for 1" pitch 2 deg. for tooth Nos. 16.6 and 204 | Faces, | .45 | .31 |
| | Flanks, | .63 | 2 69 |

In making gear patterns, turn the wooden blanks at the ends of the teeth, to the lines b i l and d v q, perpendicular to E O, and of sufficient width b l and d q, for the height of teeth and thickness of crown.

In applying the odontograph directly to the wooden blanks for a templet, it will be advisable to use a "radius rod" explained below, Fig. D. Its center pin must be at C or D, according to which wheel, unless the rod have a crook, which is not advisable. A pointed block just reaching out to C or to D should be tacked to the wheel, to hold the center pin. Its length, D o, from the plane of the actual pitch line should be equal to the square of the length of the radius of the pitch line of wheel being drawn, divided by the length of radius of the mate.

Thus for the above example, for 1″ pitch, we have, in inches,

| | |
|---|---|
| Point C from plane of its pitch line, . . . | .73 |
| " D " " " " . . | 31.2 |

The inner ends of the teeth are bounded by a concave surface, and the odontograph will not lie to it. A tooth templet of tin might be made from the odontograph, and applied. But the handier way will perhaps be found in a waxed thread attached to a center pin at O, and stretched by hand along any tooth while dressing it to the outer curves at E, laid out by the *Odontograph.* This insures a proper direction to the surface lines of the teeth.

### FOR REPEATED SETTINGS ON THE SAME WHEEL.

1. *With* "*Sliding circles.*"—With the odontograph once in correct position as in Fig. A for either *Face* or *Flank*, besides drawing the one tooth curve, pass the pencil or scriber around the point and heel of the instrument. Then

circles may be drawn through these points. These are called sliding circles, because the instrument can be slid along them to different teeth, and preserve proper position.

2. *By a "Radius Rod."*—Screws may be put through the holes in the instrument, mounting it upon the wooden rod. Mount it so that a pin or an awl put through it into the *center* of the wheel will make the instrument swing to each tooth right for marking its tooth curve.

The rules apply either for face or flank; but of course the tooth thickness must first be all laid off on the pitch line.

The most convenient radius rod for practice consists of a piece of wood about $\frac{3}{4}''$ square and a little longer than the radius of the wheel, shown in side view thus,

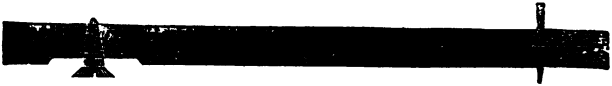


Fig. D.

where A is a common wood screw passing through one of the holes in the odontograph into the rod. The rod is cut away for about half an inch each side of A, to widen the bearing points for insuring stability of instrument on rod. At B is the awl, or center pin passed through the rod.

To use this ; 1st, by aid of the settings, lay down the odontograph in position on the wheel, or on a drawing for the purpose. Then draw the scriber around the point and heel, giving two check marks, by means of which the instrument, when removed, may readily be returned to position. Also find the exact center of the wheel and introduce the center-pin.

Next put the radius rod upon the odontograph and bring the screw A up to a moderate pressure. Then put the odontograph and its rod back to position on the wheel, and while holding the instrument firmly to its checks, swing the rod around against the center-pin and mark its touching point. Square this mark around the rod, and here pierce it with the hole that is to receive the center pin.

Now place the rod in position on the center pin, and bring the odontograph to position by its checks, pressing it, if necessary, against its moderate friction of screw pressure A. When exactly right, lift the instrument and rod; and with a screw-driver, turn up the screw A firmly.

Now all is ready for tracing the faces for one side of all the teeth (perchance the instrument is set for faces). For the faces of the other sides of these teeth, this radius rod is right, and it is only necessary to remove A, turn over the instrument and tighten A again. One of the old check-marks will serve for giving correct position. Similarly proceed for flanks.

The rod may be made of pine wood at the moment when wanted. This, together with the pin and screw, constitute a device so simple, efficient, and easily procured, that a costly telescoping metallic rod is not sent out with the instrument.

3. *Other Devices* may be employed, such as a segmental block fitting along

on several tooth points, in case the latter are turned off truly, to which the instrument may be attached with screws and thus with the block, moved around to different teeth.

Radial flanks do not require the odontograph.

### FOR NAMES AND TERMS.

The "working depth" is the actual depth to which the teeth interlock when in action.

The "Pitch Circle" usually lies at half the *working* depth of the teeth. For a pair of wheels in action the pitch circles just touch each other.

The "Pitch" is the distance on pitch line from center to center of teeth. It is equal to the circumference of the pitch circle divided by the number of teeth. This may be called circumferential pitch, but it is usually designated by simply "Pitch."

The "*chord pitch*" is the distance on a straight line from the center of one tooth at the pitch line to the center of the next.

It is always a little less than the actual circumferential pitch, the difference being greatest for wheels with few teeth. The only use for the chord pitch is for setting the dividers in the first trial at spacing around the wheels.

The "Diametrical Pitch" is the diameter of the pitch circle divided by the number of teeth, a pitch, however, that is but rarely used in connection with cast gears or gear patterns.

The "Face" and "Flank," see Fig. A.

A "Tangent" is a line just touching a circle, perpendicular to a radius. See Fig. A.

### FOR DETAILS OF TEETH.

1. *Dimensions of Teeth.*—For ordinary gear patterns for cast-iron, a well-known rule is for circumferential pitch.

| | | | |
|---|---|---|---|
| Thickness of Tooth, | 5-11 | the | Pitch. |
| " of Space, | 6-11 | " | " |
| Height of Face, | 3-10 | " | " |
| Depth of Flank, | 4-10 | " | " |
| Thickness of Crown, | 6-11 | " | " |

A good rule for "Diametrical Pitch" is, The diameter of wheel, outside to outside of points of teeth = Diameter of Pitch Circle plus twice the Diametrical Pitch. Working Depth of Teeth = twice the Diametrical Pitch. Thickness of Tooth = $\frac{3}{4}$ Working Depth. This allows 1-22 part of the Circumferential Pitch for side clearance. Allow bottom clearance = 1-16 to 1-8, average 1-10 the working depth, 1-13 by Brown and Sharpe.

For side clearance make the tooth thinner than half the circumferential pitch, by half the bottom clearance.

2. To increase the strength, strike in quarter-circle fillets at the roots of the teeth as shown in Fig. G. The radius for this may be assumed from $\frac{1}{4}$ to $\frac{1}{2}$ the depth of tooth flank.

The fillets may be much greater than just stated; greater, indeed, than usually supposed, as shown by Fig. E for radial flanks. Radial flanks are chosen for the example, because, for these, the points *a* of the teeth are most prominent, and sweep through larger spaces between fillets. Still larger spaces are required for the rack and pinion, but not so large as to interfere with the single circle fillet line. The bottom clearance is, however, considerable in Fig. E, it being 1-10 the pitch according to proportions given above, under "*Dimensions of Teeth.*"

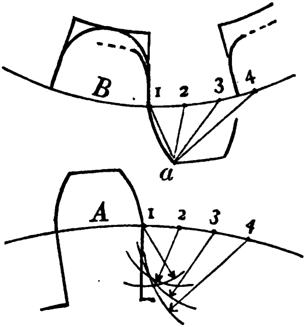


FIG. E.
Maximum Fillets.

To find the possible fillets, that is, such as the points *a* will but just clear, take spacing dividers and step off points, 1, 2, 3, 4, etc., in equal steps on both A and B. Then take a 1 for a radius from B and strike in a circle arc from 1, on A, as a center. Then take a 2 as a radius and strike in a circle with 2, on A, as a center; proceed likewise for 3, 4, etc., till the circles come up tangent with the flank of A. These various circle arcs, and their radii, are shown on A. Then a curve struck in, just touching these circles inside, is the fullest possible fillet line: this curve is called the clearing curve.—MARVIN.

The tooth *a* is shown with these clearing curves drawn in as distinguished by ending in dotted lines. Hence it appears that the bottom of the space, as shown at B, may safely be limited by a single circle arc drawn in tangent to the ungarnished flanks and bottom line.

3. By whatever method of drawing tooth-curves, the "spacing" for the teeth is as important as the profile of tooth. The greatest care should be exercised in finding the tooth centers by spacing, and also in laying off the half thicknesses.

4. Also the proper relation of the diameters, in any particular pair of wheels, is important. The diameter of the pitch line of one wheel divided by the diameter of the other, must invari-

ably be precisely equal to the number of teeth in the one wheel, divided by the number in the other. This relation is unavoidable, as is seen from the fact that if the pinions be made a little too large the teeth when in action will have contacts only on one side of the line of centers. This is evident on looking at a drawing of a pair of gears in mesh. The wear will be one-sided at first, but finally come down to contacts between teeth on both sides of the line of centers. If now the teeth have true working shapes (which is doubtful), it will be found that the pitch lines have shifted into the exact relation of the tooth numbers. That is, contacts on both sides of the line of centers are only possible with equal circumferential pitches, and not chord pitches.

### THE RESULTS OBTAINED BY THE TEMPLET ODONTOGRAPH.

Most designers are willing to take for a face curve to a tooth the closest fitting circle arc. Though the curve of the templet odontograph is not an epicycloid, yet a glance at it will suffice to show that it will fit the epicycloid much closer than is possible for a circle arc: and not only does the curve approximate with surprising closeness for curvature, but the position of the curve is also unexpectedly accurate. This latter is due to the fact that the instrument is in reality a sort of a curvilinear ruler set to position somewhat as is a draughtsman's triangle against his T-square. The advantages thus presented in the templet odontograph are made the more apparent by the following.

In accurately laying out teeth, one of two methods is usually employed, when an odontograph is not at hand. 1. By carefully drawing the rolling circle in several positions, spacing, etc., in the well-known way. 2. By employing "pitch" and "describing" templets.

In the first, the steps measure chords instead of arcs; and the thickness of lines, eyesight of draughtsman, etc., cause error.

In the second, there is liability to slipping of templets; uncertainty in placing the tracing point, etc.

In either case the result is simply a drawing of the curve. A scribe templet is then to be dressed up to fit it by an eye-and-hand operation. This brings us to where we start with the "templet odontograph."

Curves constructed for the pattern shop with the rolling circle have been reviewed with the templet odontograph. Gross differences thus detected were only accounted for by repeating the construction with greater care. This reveals the fact that the rolling circle method for gearing is too complex to be perfectly reliable. And not only this, but a knowledge of the theory of gearing, a drawing board, a set of draughting instruments, and considerable time is required.

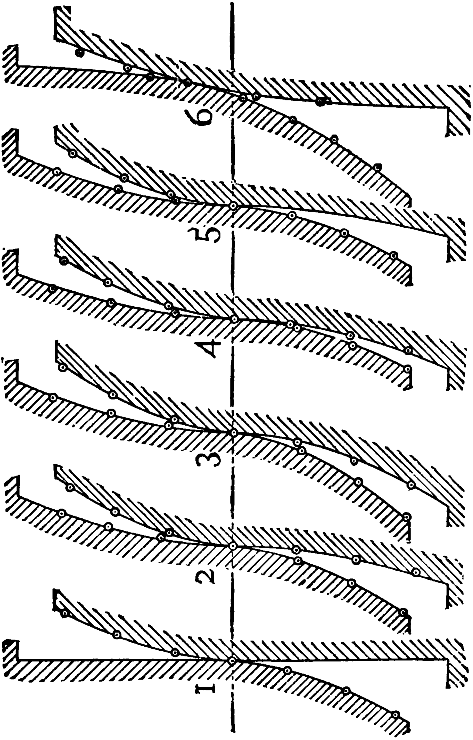
Indeed, many designers of gearing will guess at the tooth-curves, even if the theory is understood, and then whittle out and apply a guess templet, rather than take the trouble to delineate the teeth.

But in the odontograph we find a system which quickly substitutes results of mathematical calculation for the guessing and whittling, and gives in return the best attainable result.

Fig. F.

Odontograph Tooth-Curves Compared with Theory

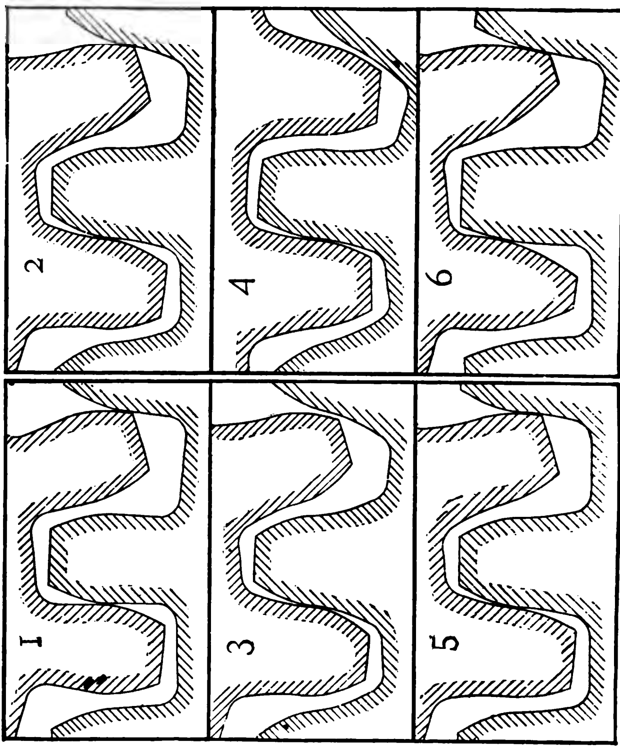
Pitch 3,044 inches.



**Full lines are by odontograph : circle dots by theory See six examples above worked out and results tabulated; wheel with 56 teeth and pinion with 16 teeth.**

Fig. G.

Tooth Outlines from the Tooth-Curves of Fig. F, by the Tabular Method.



## MODIFIED FORM OF THE WILLIS ODONTOGRAPH.

The odontograph now under consideration is termed a modified form of the Willis odontograph, not because it bears any resemblance to the Willis odontograph, but because it is based upon the theory of Prof. Willis for the forming of teeth of wheels by circle arcs, and also because it gives centers to tooth arcs, as does the Willis odontograph, instead of forming in itself a templet for the curves.

By constructing the instrument so that it can be set to various angles, instead of being fixed at 75°, and by preparing suitable tables to accompany it, we may be able with it to lay out teeth with straight flanks for all wheels, the faces of the teeth being circular arcs whose centers are given by the instrument.

The analysis by which these facts are demonstrated is essentially that of Prof. Willis, and given in his *Principles of Mechanism*, 2d ed., pp. 132, 136.

According to the Willis theory for forming each tooth with four circular arcs, whose centers are on the line of action, if certain centers be removed to an infinite distance, the arcs struck from such centers will be straight lines, perpendicular to the line of action. This is supposed to be done in the present case for the flank arcs, and when the line of action is suitably inclined, these perpendiculars become radii on the pinion and inclined lines on the wheel.

With these conditions imposed at the outset, the analysis becomes extremely simple, as follows: In Fig. 9 let A and B represent the pitch circles, C the pitch point, and E F the line of action. Draw A E and B F perpendicular to E F as shown. Let $\alpha$ = angle C A E = C B F, A C = R, B C = $r$, and C G = D. Now Prof. Willis' diagram, p. 135, becomes Fig. 9 when A $k$ and B K are made parallel to line of action as in Fig. 9. I will simply explain Fig. 9, referring those who wish to study the diagram further to Prof. Willis' book. G is a center for describing a tooth face of A, and H a center for a face of a tooth of B. The center for the flank against which the face, struck from G, works, being on B K at an infinite distance, the flank is perpendicular to E F. The point serving as the origin for this face

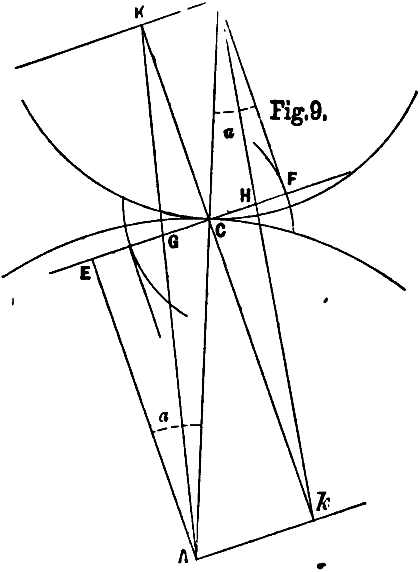


Fig. 9.

and flank may be assumed at any position* on the line E F, but should generally be not far from F, and for radial flanks for B, must be at F. Similarly for E if the flanks for E are to be radial.

To render the odontograph convenient in practical use, and to keep the radii of the tooth faces within reasonable limits for all cases, such as when the pair of wheels are of equal size and also of very different sizes, it is found advisable to make the flanks of the smaller of the pair of wheels or pinion radial in all cases; and for the larger wheel, of such inclination that the same radius for the tooth faces of the wheel will be right for the pinion. To this end C F is taken, arbitrarily equal to one-third of the pitch, so that the radius of the faces for both wheels is $G F = G C + \frac{1}{3}$ pitch. The point G is given by the odontograph, and it is easy to lay off a third of the pitch C F. Then we will have, C F B being always preserved a right angle,

---

* This fact is pointed out by Prof. Willis.

$$C\,F=\frac{P}{3}=r \sin.\ \alpha$$

$$\therefore \sin.\ \alpha=\frac{P}{3\,r}=\frac{2\,\pi}{3\,n}$$

where $n$ = number of teeth in pinion.

Again

$$C\,F:C\,H::R+r:R$$

$$C\,H=D=C\,F\frac{R}{R+r}=\frac{R}{R+r}.\frac{P}{3}$$

But

$$C\,F:C\,E::r:R$$

and

$$C\,G:C\,E::r:R+r$$

$$\therefore C\,G=C\,E\,\frac{r}{R+r}=C\,F\,\frac{r}{R+r}.\frac{R}{r}=\frac{R}{R+r}.\frac{P}{3}$$

or

$$C\,G=C\,H.$$

This shows that it is only necessary to set the odontograph once to obtain either C G or C H. Taking the distance from C to the origin of tooth arcs the same for both wheels give equal radii for the faces of both wheels, and the arcs and flanks are easily struck in as shown in Fig. 9. To draw all other arcs the centers may be found, which lie in circles concentric with the pitch lines. The drawing of the non-radial flanks of A will require a little attention that they be drawn at the proper angle with the radius, or rather simply perpendicular to E F at the proper point.

The quantities $\alpha$ and $\frac{D}{P}$ are given in terms of $n$ and $\frac{R}{r}$, respectively, in the following table for a number of values, which will aid in the practical use of the instrument, the necessary setting values being thus obtained beforehand. When the data fall between values in the table, interpolation will be necessary, though readily made.

## THE INSTRUMENT, AND MODE OF USING IT.

The modified odontograph is shown full size in Fig. 10, with its scales, &c.,

ready for use in delineating teeth of six inches pitch.

### TABLE V.

### FOR THE MODIFIED WILLIS ODONTOGRAPH.

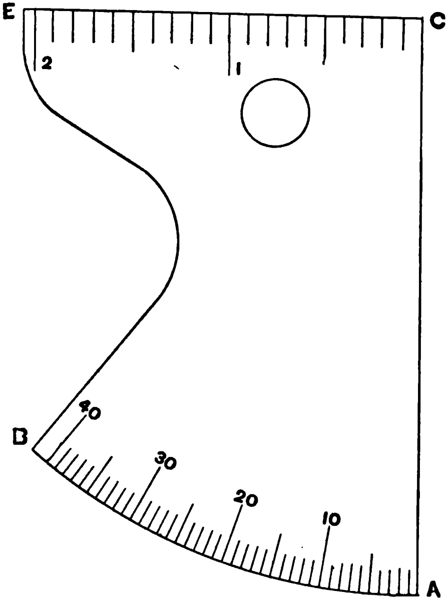
*Values of the setting angle* $\alpha$*, and of* $\frac{D}{P}$ *for position of arc center,* for tooth face for radial flanks for *pinion* and for inclined flanks for the wheel.

| Number of teeth in pinion, $=n$. | Setting angle for odontograph, $=\alpha$ | | Larger radius divided by smaller, $=\frac{R}{r}$ | Tooth center distance divided by pitch, $=\frac{D}{P}$ |
|---|---|---|---|---|
| | Deg. | Min. | | |
| 10 | 12 | 05 | 1 | .167 |
| 15 | 8 | 02 | 1.5 | .200 |
| 20 | 6 | 01 | 2 | .222 |
| 30 | 4 | 00 | 3 | .250 |
| 40 | 3 | 00 | 4 | .267 |
| 60 | 2 | 00 | 5 | .278 |
| 80 | 1 | 30 | 6 | .286 |
| 100 | 1 | 12 | 7 | .292 |
| 130 | 0 | 55 | 8 | .296 |
| 160 | 0 | 45 | 9 | .300 |
| 200 | 0 | 36 | 10 | .303 |
| 260 | 0 | 28 | 11 | .306 |
| 300 | 0 | 24 | 12 | .308 |

N. B.—The two parts of this table are entirely independent of each other, the first giving the setting angle $\alpha$, without regard to where the line is in second part of table giving $\frac{D}{P}$.

In using the instrument the point C is to be brought to the pitch point C, Fig. 9. Then the angle $\alpha$ obtained from the table is to be looked out on the sectoral scale A B, and that point placed upon the radius A C, Fig. 9. The instrument is then in position. A line should then be drawn along E C and produced in the direction of F, perpendicular to which the flanks are to be drawn. Then the distance D, obtained by multiplying $\frac{D}{P}$ of the last table by P, is to be set off on the scale C E, Fig. 10, which point is the center G, Fig. 9, from which to strike the tooth face. Then C F, equal $\frac{1}{3}$ P as explained above, is set off on line E F as the origin of a tooth face and flank as shown in Fig. 9. We then have the length G F for the radius

Fig. 10.



Modified Willis Odontograph.

Full Size.

of a face of A, which is also the radius for a face of B to be used at the centers H and G respectively. The flanks are perpendicular to E F and tangent to the tooth arcs as shown, and each face works with its tangent flank, as shown in Fig. 9. By repeating these faces and flanks the teeth become fully drawn, as in Fig. 13. The flanks of the smaller wheel are always radial, but those of the larger wheel never, except in case both wheels become equal, when the flanks of both are radial.

It will hardly be necessary to follow this description with an example.

### ODONTOGRAPH TEETH COMPARED.

Fig. 11 is an accurate drawing of a part of the teeth of a pair of wheels as laid out by the well known *Willis Odontograph.*

Fig. 12 is likewise a carefully executed drawing of a portion of the same pair of gears as laid out by the *New Odontograph.*

Fig. 13 shows the form of teeth of the

same pair of wheels as laid out by the *Modified Willis Instrument.*

The smaller of the wheels has twelve teeth, and the larger has 144 teeth.

These drawings will enable the reader to form a very correct opinion as to the relative merits of the results of the three instruments. This is not difficult to such as are familiar with the proper form of gear teeth. While this may be true, still it remains doubtful which to adopt for practical use. Such doubts, however, may be removed on taking a direct comparative view of the respective difficulties, and conveniences of application of the three instruments, which, for the purpose of furthering this object, is given below.

## PRACTICE WITH THE THREE ODONTOGRAPHS COMPARED.

The well-known *Willis Odontograph*, in laying out one side of a tooth, requires: 1st, that the pitch be laid off on the pitch line; 2d, that this pitch be bisect-

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[illegible] that a
[illegible] the pitch circle
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a quantity looked out from a table; 4th, that this be multiplied by the pitch by which the instrument is properly set on the tangent line; 5th, that a scribe be drawn along edge of instrument for the *face;* 6th, and lastly, that a radial flank be drawn from the intersection of face with the pitch line.

The *Modified Willis Odontograph* requires: 1st, that a radius be drawn; 2d, that a setting angle be found from a table by which the instrument is set; 3d, that a second quantity be taken from a table; 4th, that this quantity be multiplied by the pitch, by which the center point for the face is found and noted; 5th, that a line be drawn along the edge of the odontograph, perpendicular to which the flank is afterward drawn; 6th, that the one-third pitch be laid off rom the end of the radius upon which ne instrument was set; 7th, that by aid f a pair of dividers the face arc be ck; 8th, and finally, that the flank drawn perpendicular to the line traced the purpose as above described.

## COMPARATIVE VIEW OF THE INSTRUMENTS AND TABLES.

As to the tables for the three instruments, those for the modified Willis odontograph are the most compact of all, having only four vertical columns and fourteen horizontal, or fifty-six blocks, including everything. The table for the Willis odontograph, as published in his book,* has nine vertical columns and twenty-eight horizontal, or 250 blocks for figures. As printed on the card which accompanies the brass instrument, it has fourteen vertical columns and thirty horizontal ones, or 420 blocks for figures. The tables for the new instrument, that for radial flanks and epicycloidal teeth, has fourteen vertical columns and twenty-six horizontal ones, or 304 blocks for figures; or adding those for table for internal gearing, we get 353. It is believed, also, that Table I might be reduced by cutting out part of the horizontal columns, and still the gaps to be filled in practice, by interpolation, be no greater than in the tables for the Willis instrument. This would reduce the tables for the new odontograph, for epicycloidal and rack and pinion gearing of all ordinary forms, to about 255 blocks for figures, a number of places much less than is found in the tables on the printed card accompanying the Willis instrument, and but few more than given in the tables published in Professor Willis' book. Taking the table that accompanies the instrument, as the one which it is fair to compare with, we find the tables for the new instrument most compact.

---

* Willis, p. 187.

Comparing the instruments themselves for compactness, we see, by reference to the figures, that the modified Willis instrument is a little ahead, though either this, or the new one, may be carried under the tuck of a common pocket-book. In this regard, either of the new instruments possess a degree of portability and ready usefulness, not common to an instrument 6×14 inches, as in the older form of odontograph. In fact, the new and modified instruments might be treated as mere "pocket pieces," the accompanying tables being recorded in a note-book. This matter of compactness of tables and instruments is, however, only of secondary importance when *counterposed* with desirable results and facility of obtaining them.

These comparisons indicate that the modified Willis odontograph has advantages in the minor points of compactness of instrument and tables. But in the more essential qualities of simplicity of application, the new instrument, as above pointed out, appears to be superior in the ratio of six to eight or nine; and of form of tooth, superior in the almost mathematical correctness of the curves for faces, and in having radial flanks, as indicated by Figs. 1, 2, 11, 12, and 13. Designers of gearing, familiar with properly formed gear teeth, however much they may admire the short-cuts secured by odontographs, will undoubtedly deprecate the angularity of junction of flanks and faces found in the Willis tooth, and the rude approximation to correct tooth curves inseparable from the circle arc. The excessively thick base of tooth found in Figs. 1, 11, and 13 are undesirable features.

Fig. 12, as compared with Figs. 11 and 13, shows superiority in form of tooth, which, by those who advocate the common form of radial flanks, at least, cannot be questioned. This superiority of form of tooth, added to the compactness of instrument and especially to the important advantage possessed by it of great facility of application, will, it is believed, merit for the new odontograph decided preferences.

FIG. 11.

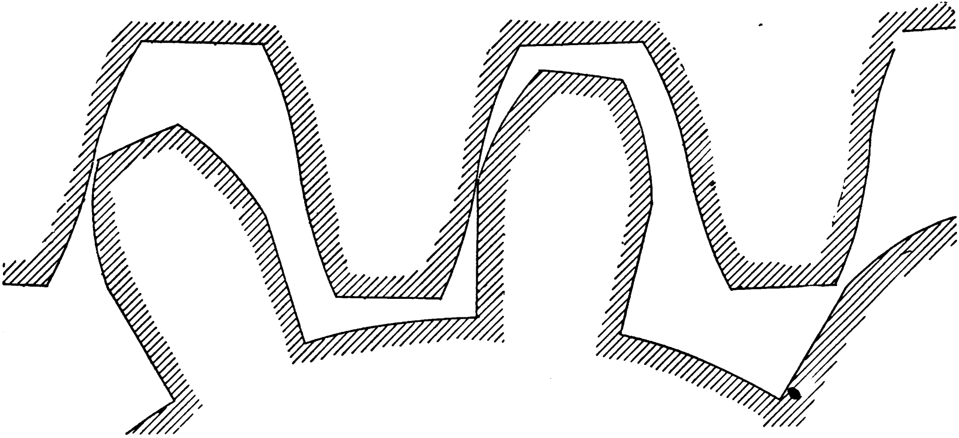


**Fig. 12.**

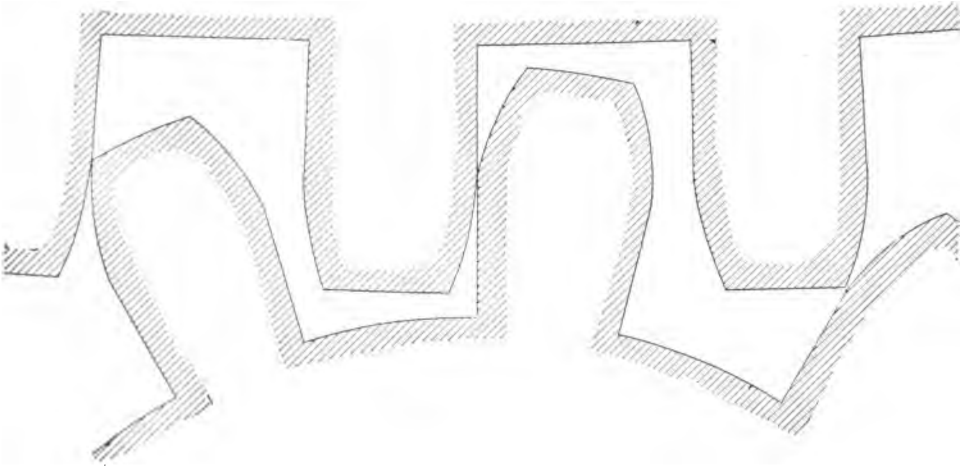
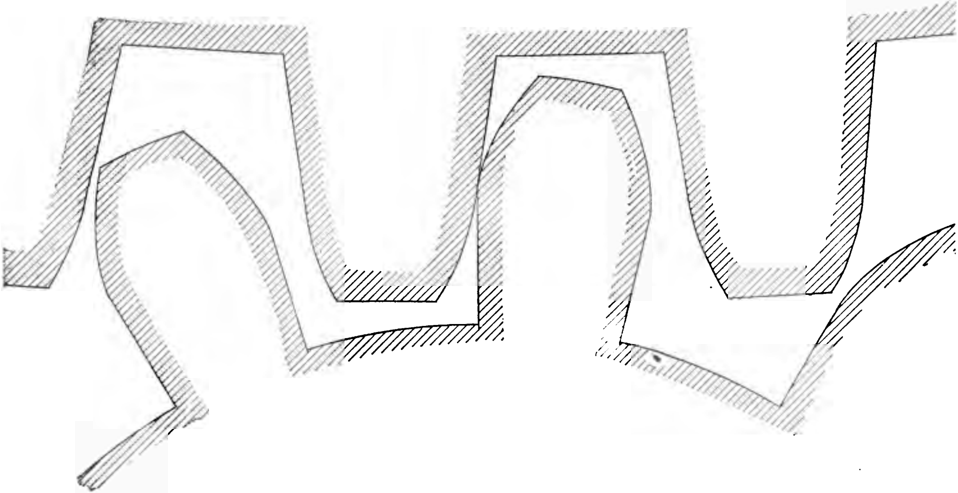


FIG. 13.



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